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# Simultaneous Backward Reduction algorithm for disassembly lot-sizing under random ordering lead time

Ilhem Slama<sup>1</sup>, Taha Arbaoui<sup>2</sup>, Faicel Hnaïen<sup>1</sup>, Oussama Ben-Ammar<sup>3</sup>, Belgacem Bettayeb<sup>4</sup> and, Alexandre Dolgui<sup>5</sup>

**Abstract**—In order to meet item demands, end-of-life (EOL) product and subassembly ordering and disassembly schedules are determined by disassembly lot sizing, which is the subject of this study. We take into consideration a stochastic version with undetermined ordering lead time (OLT). In this case, OLT stands for the amount of time that passes between placing and receiving an order (we can only order EOL products). Throughout the planning horizon, scenarios are used to model the stochasticity. The objective is to reduce the expected total of setup, purchasing, inventory, and backlog expenses. This is achieved by expressing the problem as a two-stage mixed integer linear programming (2S-MILP) model across all potential scenarios. The 2S-MILP is unsolvable since it is predicated on every scenario conceivable. A Simultaneous Backward Reduction approach is proposed to make it tractable. To confirm the suggested method’s efficacy, it is assessed in a variety of environments.

## I. INTRODUCTION AND LITERATURE REVIEW

For the three primary pillars of social equity, environmental preservation, and economic viability, sustainable development has evolved into a philosophical idea for the planet’s future. Environmental protection is one of the most important pillars to focus on because of the serious environmental degradation caused by things like excessive resource usage and global warming. [1].

One of the new problems for sustainable development is how to handle end-of-life (EOL) items more responsibly from an economic and ecological standpoint. [2]. In general, reverse logistics frameworks that include collection, disassembly, inspection, recycling, repackaging, and other methods can be used to recover EOL products. [3].

This study focuses on reverse logistics tasks such as separating reusable parts/sub-assemblies, disassembling the product into parts to recover resources, etc. [4]. Decision problems in disassembly systems fall into two categories: design problems, which include lot-sizing,

scheduling, and process planning, and operational problems, which include disassembly line balance and design for disassembly. Keep in mind that one of the most important tasks in reverse logistics is disassembly, particularly when it comes to product/material recovery like recycling and refurbishment.

One issue with disassembly production planning decisions is the disassembly lot-sizing problem, which is addressed in this work. In order to meet the needs of its parts, it seeks to ascertain how much of an EOL product and its subassemblies need to be disassembled [2].

There are several variations of the disassembly lot-sizing problem that depend on the disassembly system (two or multi-level bill of materials) and the planning horizon (one or multiple periods). The majority of material on disassembled lot-sizing systems deals with the deterministic uncapacitated problem. [5], [6], [7], [8], [9], Although a limited amount of research concentrated on the deterministic capacitated problem [10], [11], [3], [2]. For the existing surveys, readers can refer to [12].

In the actual world of industry, the disassembly process is susceptible to several unknowns, including lead times, disassembly yield, and uncertain component demand [13]. These ambiguities cause delays in the disassembly schedule and result in unfulfilled client demand.

Many scholars have employed stochastic strategies to address the disassembly lot sizing issue. To contextualize our work within the current literature, we review studies on stochastic disassembly lot-sizing problems under uncertainty of demand, yield, and disassembly lead time. Table I compiles the literature on the topic in question and includes details on the planning horizon, uncertainty kind, resolution technique, and disassembly system.

## II. PROBLEM DESCRIPTION

One way to think about the current industrial framework would be to think of a computer maker that offers end-of-life customers the option to return their outdated devices. Additionally, this company responds to the demand for reconditioned parts from its most ecologically concerned customers by maintaining a disassembly and refurbishment facility. This configuration is shown in Fig. 1.

Old computers (shown by boxes marked 1) need to be ordered, gathered, and delivered to the company’s remanufacturing factory to meet the demand for remanufactured parts. After that, they need to be sorted and

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TABLE I: Summary of relevant literature under uncertainty

Authors	Resolution	Uncertainty				#Periods	#Levels
		Yield	Demand	DLT	OLT		
[14]	Heuristics	✓				1	2
[15]	Fuzzy goal programming	✓				1	2
[16]	MILP, Lagrangian heuristics		✓			Multi	2
[17]	2S-MILP		✓			Multi	Multi
[18]	Lagrangian heuristics	✓				1	2
[19]	MILP, Lagrangian heuristics		✓			Multi	Multi
[20]	MS-MILP, Outer-approximation	✓	✓			Multi	2
[21]	MS-MILP		✓			Multi	Multi
[22]	S-LP				✓	Multi	2
[13]	Analytical model, Newsboy formulae			✓		1	2
[23]	SAA			✓		Multi	2
[24]	Analytical model, Newsboy formulae			✓		1	2
[25]	GA			✓		Multi	2
[26]	Aggregate scenario			✓		Multi	2
[27]	2S-MILP				✓	Multi	Multi
[28]	2S-MILP	✓				Multi	Multi
<b>Current paper</b>	<b>SBR</b> algorithm				✓	<b>Multi</b>	<b>Multi</b>

“MILP: Mixed Integer Linear Programming, SAA: Sample average approximation, GA: Genetic algorithm, 2S-MILP: Two stage-MIL, MS-MILP: Multi stage-MILP, SBR: Simultaneous Backward Reductio.”

dismantled to extract usable parts for the remanufacturing process (the parts are shown by the circles with the letters a, b, and c).

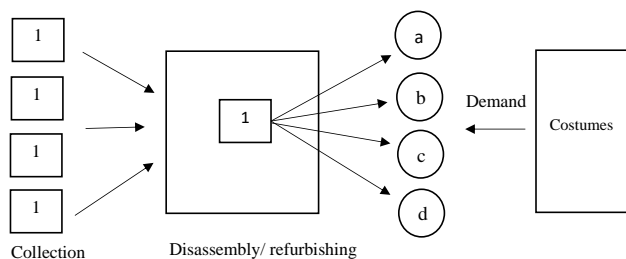


Fig. 1: Remanufacturing from collection to disassembly

In actuality, there is no set amount of time that passes between placing an order and actually receiving an EOL product. The primary reason for this uncertainty is the lack of EOL solutions available on the market. As a result, it is assumed in this study that there is a root item shortage, meaning that end-of-life products are unavailable for order at any time. A distinct set of scenarios is established in order to include this uncertainty in the decision-making process. It is anticipated that there will be a need for parts and that disassembling EOL items will take some time. Lastly, the capacity for disassembly is supposedly infinite.

We take into account time-varying purchasing costs based on the state of the market, which implies that the costs of purchases could change based on various planning periods. The necessary equipment needs to be assembled once each piece has been disassembled. There is a setup fee as a result. Additionally, items stored to

meet future demand result in inventory holding costs.

Backlog and inventory holding costs are associated with the risk of unclear ordering lead times. As a matter of fact, if the EOL products are not delivered on schedule, the disassembly and refurbishing process does not begin as planned, which results in backlog costs. Similarly, if the EOL product is received ahead of schedule, an inventory cost is incurred.

This study examines the case of a multi-echelon disassembly system. As seen in Fig. 2, an EOL product to be ordered and disassembled is represented by a root item, a sub-assembly to be further disassembled is represented by an intermediate item (items 2–6), and a component that cannot be dismantled is represented by a leaf item (items a–d). The number of components that were obtained through the disassembly processes is indicated in parentheses.

Customers have a certain desire for disassembled things at all times, as was previously noted. For leaf items, this need is external; for root and sub-assembly items, it is internal. Ultimately, the only way to meet the demand is to disassemble the EOL goods.

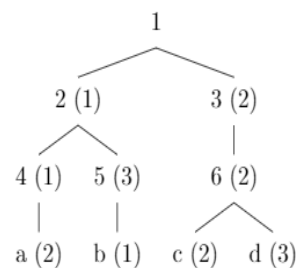


Fig. 2: Multi-echelon disassembly lot-sizing system

This study is supposing:

- 1) After a random ordering lead time, the EOL product is available for disassembly;
- 2) The ordering lead time is a discrete random variable confined by the known intervals  $OLT_t^-$  and  $OLT_t^+$ , with a known probability distribution;

The issue addressed in this work is the difficulty of figuring out, under random OLT, over a specific planning horizon, the ordering schedules of the EOL and the disassembly schedules of all parent objects in the provided disassembly system. The goal is to reduce the total anticipated expenses for setup, purchases, inventory holding, and backlog.

### III. LINEAR STOCHASTIC MODEL

The topological order is taken into account for all components in the issue formulation examined in this research, both left to right and bottom to top. Let  $N$  be the index of the final item found after disassembling it, and let  $i = 1$  be the index of the root item. Allow the subsequent trio of sets: (i)  $\mathcal{I}$ , the set of items  $i$  with  $i = 1, 2, \dots, i_{l-1}, i_l, \dots, N$ , where  $i_l$  denote the index for the first leaf item, (ii)  $\mathcal{I}_c$ , the set of items  $i$  of the last level of product structure that can no longer be disassembled. These items verify the following equality:  $\forall i \in \mathcal{I}_c, \forall j \in \mathcal{I} \mid a_{ji} = 0$  and  $\mathcal{I}_c = \{i_l, \dots, N\}$ , and (iii)  $\mathcal{I}_e$ , the set of remaining items such as  $\mathcal{I}_e = \{2, \dots, i_{l-1}\}$ . The full list of notations used throughout this paper is given in Table II.

In this work, we present a definition of stochastic optimization based on scenarios. For the EOL product, a scenario shows a potential realization of the ordering lead time in each period.

*Definition 3.1:* Let the collection of all conceivable scenarios be  $\Omega$ . A realisation of the ordering lead times from periods 1 to  $t$  corresponds to each scenario  $\omega$ . More specifically, in scenario  $\omega$ , the EOL product is accessible in period  $t + OLT_t^\omega$  if any purchase orders of EOL product start in period  $t$ . The probability of scenario  $\omega$  is  $P_\omega$ . If the ordering lead times in each period are independent,  $P_\omega = \prod_{t \in \mathcal{T}} P(OLT_t = OLT_t^\omega)$ ,  $\sum_{\omega \in \Omega} P_\omega = 1, \forall \omega \in \Omega$  and  $|\Omega| = \prod_{t \in \mathcal{T}} (OLT_t^+ - OLT_t^- + 1)$

An example of a two-stage multi-period stochastic mixed-integer program that can describe the researched problem is 2S-MILP. The choices made in period 0 before seeing the OLT are reflected in the first-stage variables. These choices concern the quantity of setup and disassembly for item  $i \in \mathcal{I}_e$ , as well as the quantity of EOL product procurement for period  $t$ . The inventory levels of item  $i \in \mathcal{I}$  and the backlog levels of item  $i \in \mathcal{I}_c$  at the end of period  $t$  correspond to the second stage variables, often known as recourse decisions. Following the realization of the OLT for every scenario  $\omega$ , these choices are made.

TABLE II: Notation

<i>Index</i>	
$t$	Index of period $t, t \in \mathcal{T}$
$i$	Index of item $i, i \in \mathcal{I}$
$\omega$	Index for scenarios $\omega$ of ordering lead times, $\omega \in \Omega$ .
<i>Parameters</i>	
$\mathcal{T}$	Set of time periods of the planning horizon
$\mathcal{I}$	Set of items
$\mathcal{I}_c$	Set of leaf items
$\mathcal{I}_e$	Set of sub-assembly items
$\Omega$	Set of possible scenarios of lead times
$d_{it}$	External demand for item $i$ in time period $t, \forall i \in \mathcal{I}_c$
$a_{ji}$	Number of units of item $i$ obtained from disassembling $j$
$I_{i0}$	Initial inventory for each item
$OLT_t^\omega$	Random ordering lead time in time period $t$ at scenario $\omega$
$h_i$	Per-period inventory holding cost of one unit of item $i, \forall i \in \mathcal{I}$
$s_i$	Per-period setup cost of parent item $i, \forall i \in \mathcal{I}_e$
$b_i$	Per-period backlog cost of one unit of item $i, \forall i \in \mathcal{I}_c$
$c_t$	Purchase cost of EOL product in time period $t$
$\phi_i$	Parent of item $i, \forall i \in \mathcal{I}_{\setminus\{1\}}$
$M$	A large number
<i>Functions</i>	
$\mathbb{E}(\cdot)$	Expected value
$P_\omega$	Probability value for each scenario $\omega$
<i>Decision variables</i>	
$Q_{it}$	Quantity of parent item $i$ to disassemble in period $t$
$Z_t$	Quantity of EOL product to procure in time-period $t$
$Y_{it}$	Binary indicator of disassembly for item $i$ in period $t$
<i>Variables</i>	
$I_{it}^\omega$	Inventory level of item $i$ at the end of period $t$ of scenario $\omega, \forall i \in \mathcal{I}$
$B_{it}^\omega$	Stockout level of item $i$ at the end of period $t$ of scenario $\omega, \forall i \in \mathcal{I}_c$

The set  $\Omega$  of all conceivable OLT situations and the probability  $P_\omega$  of each scenario  $\omega$  constitute the basis of the stochastic formulation of the multi-level disassembly lot-sizing problem. The primary goals are to choose the proper ordering and disassembly strategy and reduce the expected objective value.

Our model is a development of [27]'s work in general. It can be described using the 2S-MILP model that follows:

$$\mathbb{E}(TC) = \min \sum_{t \in \mathcal{T}} \left( \sum_{\omega \in \Omega} P_\omega \left( \sum_{i \in \mathcal{I}} h_i \cdot I_{it}^\omega + \sum_{i \in \mathcal{I}_c} b_i \cdot B_{it}^\omega \right) + Z_t \cdot c_t + \sum_{i \in \mathcal{I}_e} s_i \cdot Y_{it} \right) \quad (1)$$

$$I_{1t}^\omega = \sum_{\tau \in \mathcal{A}_i^\omega} Z_\tau - \sum_{\tau=1}^t Q_{1\tau} + I_{10} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (2)$$

$$I_{it}^\omega = \sum_{j \in \phi(i)} a_{ji} \cdot Q_{j\tau} + I_{i0} - \sum_{\tau=1}^t Q_{i\tau} \quad \forall i \in \mathcal{I}_e, \forall t \in \mathcal{T} \quad (3)$$

$$I_{it}^\omega - B_{it}^\omega = \sum_{j \in \phi(i)} a_{ji} \cdot Q_{j\tau} + I_{i0} - \sum_{\tau=1}^t d_{i\tau} \quad \forall i \in \mathcal{I}_c, \forall t \in \mathcal{T} \quad (4)$$

$$Q_{it} \leq Y_{it} M \quad \forall i \in \mathcal{I}_e, \forall t \in \mathcal{T} \quad (5)$$

$$Q_{it} \geq 0 \quad \forall i \in \mathcal{I}_e, \forall t \in \mathcal{T} \quad (6)$$

$$Y_{it} \in \{0, 1\} \quad \forall i \in \mathcal{I}_e, \forall t \in \mathcal{T} \quad (7)$$

$$I_{it}^\omega \geq 0 \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (8)$$

$$B_{it}^\omega \geq 0 \quad \forall i \in \mathcal{I}_c, \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (9)$$

$$Z_t \geq 0 \quad \forall t \in \mathcal{T} \quad (10)$$

Over the planning horizon, the goal function (Eq.1) seeks to minimize the total of the allowances for inventory holding, backlog, purchasing expenses, and setup costs. For a scenario  $\omega$ , the inventory balance level for the EOL product, each sub-assembly item, and each leaf item  $i$  is defined by constraints (2), (3), and (4), respectively, at the end of each time period  $t$ . The set of periods whose disassembled parent items are available is denoted by  $\mathcal{A}_i^\omega := \{\tau \in \mathcal{T} \mid \tau + OLT_\tau^\omega \leq t\}$ . If any disassembly work needs to be done within a certain time frame, constraints (5) ensure that a setup cost is generated in that time frame. 6–10 constraints offer the parameters that surround the choice variables.

#### IV. SIMULTANEOUS BACKWARD REDUCTION (SBR) ALGORITHM

Due to the complexity of probabilistically constrained programs, the use of approximation techniques based on scenario reduction is often accepted and justified when solving problems involving large sets of scenarios. These approximate methods make solvable models of the type (1) - (10), whose dimensions increase exponentially with the number of scenarios generated.

Theoretically, scenario reduction is an independent problem, but in most cases, it is indispensable for solving a probabilistically constrained program, whose uncertain data are described by discrete probability distributions.

Rather than randomly generating a polynomially tractable subset of scenarios, we propose to reduce the set of scenarios by eliminating those most unlikely to occur de facto, according to metrics of probabilistic distributions.

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#### Algorithm 1 Simultaneous Backward Reduction (SBR) [29]

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##### Step 1:

$$C_{ku} = C_T(\omega_k, \omega_u), \forall k, u = \{1, \dots, |\Omega|\}$$

$$C_{ll}^1 = \min_{b \neq l} C_{lb}, \forall l, l = \{1, \dots, |\Omega|\}$$

$$z_l^1 = P_l \times C_{ll}^1, \forall l, l = \{1, \dots, |\Omega|\}$$

$$l_1 \in \arg \min_{a=\{1, \dots, S\}} z_l^1, G^1 = l_1$$

##### Step i:

$$C_{kl} = \min_{b \notin G^{(a-1)} \cup \{l\}} C_{kb}, l \notin G^{(a-1)}, k \in G^{(a-1)} \cap \{l\}$$

$$z_l^a = \sum_{k \in G^{(a-1)} \cup \{l\}} p_k \times C_{kl}^a, l \notin G^{(a-1)}$$

$$l_a \in \arg \min_{l \notin G^{(a-1)}} z_l^a$$

$$G^a = G^{(a-1)} \cup \{l_a\}$$

**Step**  $|\Omega| - n + 1$  Provide the optimal redistribution of  $n$  preserved scenarios.

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The SBR algorithm starts by finding the first scenario to be eliminated, which corresponds to the scenario with the smallest distance to the others (i.e. the scenario that can be best represented by the remaining scenarios). The first iteration of this algorithm therefore consists in finding the scenario whose elimination will result in the smallest error. Then, at each subsequent iteration, the probability distribution of the remaining scenarios is updated by adding the probability value of the last scenario eliminated to that of its closest non-eliminated scenario. In this way, all  $m$  scenarios are iteratively eliminated in the same way as the first scenario.

#### V. COMPUTATIONAL EXPERIMENTS

This section presents the results of experiments conducted to study the behavior of the proposed optimization approaches. The proposed models are implemented in *C* with Concert Technology and solved with IBM CPLEX 12.5 on a PC with processor Intel (R) Core™ i7-5500 CPU @ 2.4 GHz and 8 Go RAM under Windows 10 Professional.

##### A. Numerical example

The *small size testbed* consists of an example of a multi-echelon disassembly system as presented in Fig. 3. This example deals with a disassembly system with five items and two-parent elements. The number in parenthesis represents the disassembly yield for each item. As detailed in Tables III, we considered eight test instances with a finite planning horizon with 3 to 10 periods and an ordering lead time range  $[L_i^+ - L_i^-] = [1, 2] \forall i \in \mathcal{I}_e, \forall t \in \mathcal{T}$ . Here, the ordering lead time follows the uniform probability distribution.

The demands for components over the planning horizon  $d_{it}$  are generated randomly from  $D \sim U(0, 100) \forall i \in \mathcal{I}_c$  and  $\forall t \in \mathcal{T}$ . The unit time inventory holding lost sales and backlog and costs are  $h_i = 3, \forall i \in \mathcal{I}_{\setminus \{1\}}, b_i = 2 \times h_i$  and  $e_i = 5 \times b_i, \forall i \in \mathcal{I}_c$ , respectively. The setup cost  $s_i$  is generated randomly from  $D \sim U(500, 1000) \forall i \in \mathcal{I}_e$ , where  $D \sim U(a, b)$  denotes that a discrete



uniform distribution is used to generate the parameter randomly with support  $\{a, \dots, b\}$ .

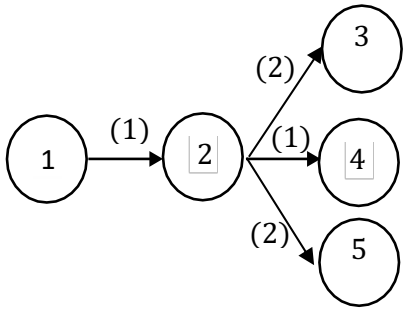


Fig. 3: Example of multi-echelon system.

We begin our experiments by comparing the performance of the suggested SBR technique to the standard stochastic 2S-MILP model. More specifically, Table III lists the total number of scenarios  $|\Omega|$ , the total number of scenarios that were aggregated into  $|\Omega^1|$ , the Gap, and the CPU times in seconds for each instance set. Here  $|\Omega^1|$  represents 30% of  $|\Omega|$ .

TABLE III: SBR Performance (CPU time).

Instances	Instance set			TS-MILP	SBR		
#Instance	$ \mathcal{N} $	$ \mathcal{T} $	$ \Omega $	CPU	$ \Omega^1 $	CPU	Gap
1	5	3	$2^3$	0.06	3	0.031	0.00
2	5	4	$2^4$	0.39	5	0.037	0.02
3	5	5	$2^5$	2.60	10	0.09	0.02
4	5	6	$2^6$	30.62	20	4.07	0.05
5	5	7	$2^7$	1470	39	6.049	0.02
6	5	8	$2^8$	-	77	7.041	0.15
7	5	9	$2^9$	-	154	11.07	0.21
8	5	10	$2^{10}$	-	307	22.51	0.22

(-): No feasible solution is available. (Gap): The percentage deviation between  $ATC$  and the expected total cost ( $\mathbb{E}(TC)$ )

As the same table illustrates, the number of all conceivable possibilities in the stochastic 2S-MILP grows exponentially with the number of periods. Therefore, occurrences with more than seven periods and two-parent items are not solved by the 2S-MILP. Contrarily, the SBR technique has a very low number of scenarios. Given a small deviations Gap (less than 0.25%), the SBR plays an important role in finding a solution very close to the optimal one in a very reasonable time.

### B. Performance analysis

Here are the test results from a major test problem we performed to demonstrate the efficiency of the 2S-MILP model and the SBR technique in general. Ten problems make up each [0] combination of three levels of the number of components (10, 20, and 30) and three levels of the number of periods [0] (5, 7, and 10). These are the huge test questions. Three ordering lead time range sizes are taken into consideration for each level of

the number of periods (1, 2 and 4). The creation process for each parameter is displayed in Table IV, where  $D \sim U(a, b)$  denotes that the parameter is created at random using a discrete uniform distribution.

An essential component of the decision-making process is computation time. We gave CPLEX a 3600 second time limit to run as a result. The following performance metrics are employed:

- CPU: Computation times, in seconds, i.e. the time(s) required to find optimal solutions;
- $I^*$ : Number of optimal (out of 10 instances) obtained by the CPLEX solver.

TABLE IV: Characteristics of data sets

Parameters	Values
$h_i$	$D \sim U(10, 30)$
$c_t$	$D \sim U(40, 60)$
$s_i$	$D \sim U(500, 1000)$
$I_{i0}$	$D \sim U(20, 100)$
$b_i$	$D \sim U(50, 100)$
$d_{it}$	$D \sim U(50, 160)$
$a_{ij}$	$D \sim U(1, 3)$
$OLT_{it}$	$D \sim U(OLT_i^-, OLT_i^+)$

Table V indicates that 2S-MILP performance is influenced by ordering lead times, number of periods, and number of items. Of these criteria, the ordering lead-time range and the number of periods have the most effects on computation time. Additionally, once the size of the uncertainty interval exceeds a certain amount (i.e.,  $OLT_t^+ - OLT_t^- \geq 2$ ), 2S-MILP is impossible to solve any case with 10 periods.

## VI. CONCLUSION

For a particular kind of end-of-life product, this study examined a multi-level disassembly lot-sizing challenge. Under the interval representation of uncertainty, the amount of time it takes to receive an EOL product following the placement of a purchase order is seen as a stochastic variable. Determining the order and quantity for disassembling the EOL product and its subassemblies is the issue. To minimize the projected total cost, an ideal two-stage multi-period stochastic mixed-integer program is proposed. This model uses the set of all conceivable situations to define the random parameter. To alleviate the scalability issues, a Simultaneous Backward Reduction algorithm (SBR) is suggested. Experimental results show the effectiveness of the proposed models and the convergence of the resulting SBR.

We are still conducting research in this area. First, creating a heuristic to handle the big test cases is an interesting project. Furthermore, the multi-stage model presents an intriguing avenue for further investigation in the context of the traditional tree representation scenario.

TABLE V: Performances of the 2S-MILP

(a) Problem with 5 periods.

2S-MILP						
#components	10		20		30	
$OLT_t^+ - OLT_t^-$	$I^*$	CPU	$I^*$	CPU	$I^*$	CPU
1	10	0.45	10	0.93	10	1.64
2	10	4.84	10	14.35	10	12.5
4	10	96.297	10	331.672	10	723.781

(b) Problem with 7 periods.

2S-MILP						
#components	10		20		30	
$OLT_t^+ - OLT_t^-$	$I^*$	CPU	$I^*$	CPU	$I^*$	CPU
1	10	4.93	10	3.64	10	13.87
2	10	499.35	10	1243.25	10	1336.56
4	0	-	0	-	0	-

(c) Problem with 10 periods.

2S-MILP						
#components	10		20		30	
$OLT_t^+ - OLT_t^-$	$I^*$	CPU	$I^*$	CPU	$I^*$	CPU
1	10	83.37	10	381.60	10	1022.46
2	0	-	0	-	0	-
4	0	-	0	-	0	-

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