Supplier Selection Considering Flexibility, Order Splitting, and Uncertainty of lead times

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Oussama Ben-Ammar[1], Belgacem Bettayeb[2], Ilhem Slama[3] and Alexandre Dolgui[4]

***Abstract*— Effective replenishment planning and inventory control are essential for the smooth operation and adaptability of supply chains. These aspects play a pivotal role in upholding a company's competitiveness and triumph in today's fiercely competitive markets. Supply chain planners encounter significant hurdles in choosing the most appropriate suppliers in diverse scenarios, reducing average inventory levels, and determining optimal safety lead times. This research tackles these challenges by examining and evaluating a multi-period replenishment planning issue within the framework of dynamic demand and multiple suppliers. The suppliers are pre-selected and defined by procurement costs, with lead times considered as independent discrete random variables with known and limited probability distributions. The goal is to optimize the distribution of order quantities among these pre-selected suppliers while minimizing the anticipated total cost. Two strategies and corresponding linear models are suggested to investigate the impact of dividing orders between suppliers, order crossover, and order flexibility. Numerical experiments provide evidence that concurrently considering splitting and flexibility yields benefits in terms of cost optimization.**

I. Introduction and related publications

Efficient supply chain management is crucial for ensuring timely deliveries and customer satisfaction. Over the years, significant attention has been devoted to addressing uncertainties in supply chains and their impact on performance. This includes strategies to mitigate supply uncertainty and cope with disruptions [1], [2], [3], [4]. However, existing inventory models often overlook dynamic demand and lead time uncertainties, which are critical in today's volatile markets, such as those exacerbated by the COVID-19 pandemic [5], [6], [7], [8].

Dealing with uncertainty in procurement strategies, particularly in single-sourcing with uncertain lead times, poses challenges. Traditional approaches like safety lead times and safety stocks have been explored, but stochastic optimization techniques offer more nuanced solutions [9], [10], [11], [12]. Multi-period supply planning under lead time uncertainty is complex due to order crossovers and probabilistic dependencies between received quantities. Existing models often oversimplify or ignore these factors, limiting their applicability [13], [14].

Research addressing uncertainty in lot-sizing problems under lead time uncertainty is limited. Some studies have proposed optimal policies but often focus on deterministic demands or single-sourcing procurement strategies [15], [16].

Supplier selection under uncertainty is another crucial aspect of supply chain management, with delivery performance consistently identified as a top priority [17], [18], [19], [20]. However, existing decision-making techniques are often static and may not adequately account for dynamic environments [21].

Dynamic supplier selection has gained attention, especially in reconfigurable supply chains, where decisions are made at tactical or operational levels. Splitting orders among multiple suppliers has been favored over single sourcing due to reduced lead time risks [22], [23], [24], [25]. Despite efforts to manage inventory under lead time uncertainty, alternative strategies warrant further investigation [26], [27].

This study advances current understanding by exploring multi-sourcing dynamic replenishment planning under lead time uncertainty. It considers allocating order quantities among pre-selected suppliers and proposes two strategies based on order splitting and flexibility. The challenge lies in balancing uncertainty-related costs and purchasing costs.

The subsequent sections of our paper are structured as follows. Section II outlines the problem under study and introduces various approaches concerning order splitting and flexibility. Section III elaborates on two stochastic models related to the splitting of demands and order flexibility. Section IV presents a numerical example demonstrating the discussed linear stochastic models. Lastly, Section V summarizes our conclusions and outlines potential future research areas related to this study.

II. Problem description

This study explores a scenario involving a single buyer and multiple suppliers, focusing on a multi-period replenishment planning issue for a specific product. Each period's demand, which is integral and predetermined, may be fulfilled by one or more chosen suppliers, with delivery following a stochastic discrete lead time. Allowances are made for tardiness (backlog), with all unmet demands assumed to be fully backordered. It is important to note that there are no restrictive assumptions regarding the overlap of orders or the demand structure (specific demand values per period). Ordering costs

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[1]Oussama Ben-Ammar is an associate professor at EuroMov Digital Health in Motion, Univ Montpellier, IMT Mines Ales, Ales, France oussama.ben-ammar@mines-ales.fr

[2]Belgacem Bettayeb is an associate professor at CESI, LINEACT EA 7527, Lille Campus, 8 boulevard Louis XIV, 59046 Lille, France bbettayeb@cesi.fr

[3]Ilhem Slama is an associate professor at Université de Technologie de Troyes, LIST3N, 12 rue Marie Curie, 10010 Troyes, France ilhem.slama@utt.fr

[4]Alexandre Dolgui is a professor at IMT Atlantique, LS2N, UMR-CNRS 6004, La Chantrerie, 4 rue Alfred Kastler, 44300 Nantes, France alexandre.dolgui@imt-atlantique.fr

are deemed insignificant or are included in the purchase costs. The available data includes period-specific demand, per-unit inventory holding and backlogging costs, a list of suppliers for the product, and their per-unit prices. As for the lead times from suppliers, the only pre-known detail is their probability distributions, which are either statistically derived or estimated by the suppliers and encompass factors like production capacity, processing, queuing, and distribution times unique to each supplier. Once a supplier is selected and the order quantity confirmed, delivery follows a random lead time based on its probability distribution. The uncertainty in lead times means that orders might be received in a different order than planned, potentially causing extended storage periods and tardiness, thereby escalating costs.

As this is a multi-period and multi-supplier problem, two strategies are plotted relative to (i) the splitting of each demand into small batches over several suppliers at different periods and (ii) flexibility. No flexibility means that, for each supplier, all quantities ordered in the same period are regrouped and delivered in the same package. Notice: the notion of order flexibility considered here is meaningless when lead times are deterministic because, in that case, all quantities released in the same period, regrouped or not, will surely be received together. If lead times are deterministic and there is neither capacity constraint nor ordering costs, the optimal solution is trivial. All demands are satisfied logically by the supplier proposing the lowest price.

In the next section, the problem of distribution of order quantities over pre-selected suppliers is modelled by considering the uncertainty of lead times, the possibility of order splitting and flexibility on orders. The modeling approach is based on stochastic linear programming with aggregated scenarios. Table I details the mathematical notations used in this paper.

TABLE I: Notations

Parameters	
$\mathcal{T}$	number of time buckets
t	index of the period
$\mathcal{S}$	set of suppliers, with $\lvert\mathcal{S}\rvert = S$
s	index of suppliers, $s = 1, ..., S$
D_t	demand of period t, an integer
c^h	unit inventory holding cost per period
c^b	unit backlog cost per period
c^s	unit purchasing cost from supplier s
$L^s_{\tau,\omega}$	lead time associated to the quantity ordered at period τ from supplier s for the scenario ω. It is a discrete random variable varying between L^s_- and L^s_+ and used for the case where demands splitting is not allowed
$L^s_{\tau,t,\omega}$	lead time associated to the quantity ordered at period τ from supplier s to satisfy demand D_t for the scenario ω. It is a discrete random variable varying between L^s_- and L^s_+ and used for the case where demands splitting is allowed
Variables	
$Q^s_{\tau,t}$	quantity ordered from supplier s at period τ to satisfy D_t, it is an **integer decision variable**, which is used for the case where demands splitting is allowed
Q^s_τ	quantity ordered from supplier s at period τ, it is an **integer decision variable**, which is used for the case where demands splitting is not allowed
$y^s_{\tau,t}$	is equal to 1 if D_t is ordered from supplier s at period τ; zero otherwise. It is **a binary decision variable**, which is used for the case where demands splitting is allowed
$A_{t,\omega}$	arrival quantity at period t for the scenario ω
I_t	inventory level at the end of period t
I^+_t	inventory to carry at the end of period t. It is equal to I_t if $I_t \geq 0$ and zero otherwise
I^-_t	backlog level at the end of period t. It is equal to $-I_t$ if $I_t \leq 0$ and zero otherwise
Functions	
$\mathbb{P}(.)$	probability value
$\mathbb{E}(.)$	expected value
$\mathbb{1}_A$	indicator function, equals 1 if event A is true; 0 otherwise

III. Linear stochastic models

This section details two stochastic models relative to demands' splitting and order flexibility. To do so, for each choice regarding flexibility, first, some sets that delimit the decision domains and the state spaces are defined. Then, the corresponding ILP formulation for the case with demands splitting policies is given.

A. Characterizing the scenarios of "No Flexibility" strategy

To account for the uncertainty in lead times, the following elements are introduced:

- the set of indices $\mathcal{M}^{NF}_t$ of all order quantities Q^s_τ used to calculate the backorder level at t
- the set of all possible scenarios Ω^0
- the set of aggregated scenarios Ω^{NF}
- the probability p_ω of each aggregated scenario $\omega \in \Omega^{NF}$

For a given period t, we can split all possible Q^s_τ (each quantity is ordered from supplier s at period τ) into three groups: (i) orders that will surely be received before the end of period t (see Fig. 1-(a) and -(b)), (ii) orders that will never be received before the end of period t ($\tau + L^s_- > t$) (Fig. 1-(e)), and (iii) orders that contain uncertainties about their arrival at period t (Fig. 1-(c) and -(d)). Note that only the order quantities belonging to the third group are subject to lead time uncertainty. Therefore, they are used to define the set $\mathcal{M}^{NF}_t$.

Definition III.1. *Let $\mathcal{M}^{NF}_t$ be the set of indices of all orders Q^s_τ that can be involved in calculating the backlogging level at period t. This set will be used in models that do not consider order flexibility. It corresponds to the two cases illustrated in sub-figures (c) and (d) of Fig. 1, and is defined by:*

$$\mathcal{M}^{NF}_t = \left\{(s,\tau) \in \mathcal{S} \times \mathcal{T} : t - L^s_+ + 1 \leq \tau \leq t - L^s_-\right\}$$

Each quantity Q^s_τ ordered at period τ from supplier s arrives after a random lead time L^s_τ. The periods in which this quantity can arrive are within the interval $[\tau + L^s_-, \tau + L^s_+]$. Therefore, the inventory level at any period τ depends on the realization of lead times of the ordered quantities indexed in the set $\mathcal{M}^{NF}_t$.

Definition III.2. *Let $\alpha^s_{\tau,\omega} = \mathbb{1}_{\{L^s_{\tau,\omega} \leq t-\tau\}} : (s,\tau) \in \mathcal{M}^{NF}_t$ be an indicator that expresses if the quantity ordered from supplier s at period τ arrives before the end of period t for*

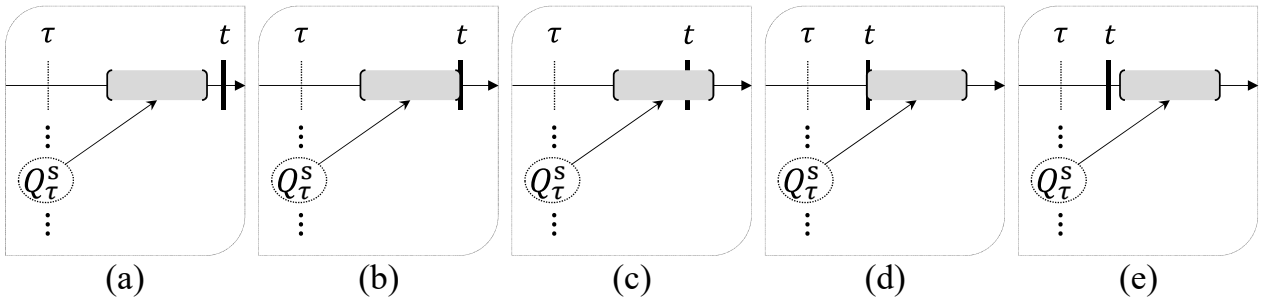

Fig. 1: Different cases of arrivals periods for NF strategy.

a given scenario ω:

$$\alpha^s_{\tau,\omega} = \begin{cases} 0 & \text{if } \tau + L^s_{\tau,\omega} > t, \quad \text{with } \mathbb{P}(L^s_{\tau,\omega} > t - \tau) \\ 1 & \text{if } \tau + L^s_{\tau,\omega} \leq t, \quad \text{with } \mathbb{P}(L^s_{\tau,\omega} \leq t - \tau) \end{cases}$$

A set of $\alpha^s_{\tau,\omega}$ for all $(s,\tau) \in \mathcal{M}^{NF}_t$ defines a given scenario ω. The set of all possible aggregated scenarios, whose cardinality $|\Omega^{NF}|$ is $2^{|\mathcal{M}^{NF}_t|}$, is defined as follows:

$$\Omega^{NF} = \left\{ \left(\alpha^s_{\tau,\omega}\right)_{(s,\tau)\in\mathcal{M}^{NF}_t} : w \in \{1, ..., 2^{|\mathcal{M}^{NF}_t|}\} \right\}$$

In the first scenario $\omega_1 \in \Omega^{NF}$, $\forall (s,\tau) \in \mathcal{M}^{NF}_t$ each quantity ordered from supplier s at period τ arrives after period t. It is a null vector whose dimension is the number of elements of $\mathcal{M}^{NF}_t$: $\omega_1 = \mathbf{0}_{|\mathcal{M}^{NF}_t|}$. In the last scenario $\omega_{[\Omega^{NF}|}$, $\forall (s,\tau) \in \mathcal{M}^{NF}_t$ each quantity ordered at period τ from supplier s arrives before period t: $\omega_{[\Omega^{NF}|} = \mathbf{1}_{|\mathcal{M}^{NF}_t|}$.

Property III.1. *Each scenario ω is characterized by its probability of occurrence p_w as defined below:*

$$p_\omega = \prod_{(s,\tau)\in\mathcal{M}^{NF}_t} \begin{pmatrix} \alpha^s_{\tau,\omega}\mathbb{P}(L^s_{\tau,\omega} \leq t-\tau) + \\ (1-\alpha^s_{\tau,\omega})\mathbb{P}(L^s_{\tau,\omega} > t-\tau) \end{pmatrix} \forall \omega \in \Omega^{NF}$$

where $\alpha^s_{\tau,\omega} = \mathbb{1}_{\{L^s_{\tau,\omega} \leq t-\tau\}}$ is a binary value, and such that $\sum_{\omega\in\Omega^{NF}} p_\omega = 1$.

Based on the aggregation strategy resulting in the sets $\mathcal{M}^{NF}_t$ and Ω^{NF}, a first stochastic model is proposed. It is denoted by M_1 and considers neither the flexibility on orders nor order splitting.

This model, noted by M_1, considers the strategy where neither order flexibility nor splitting is allowed. The corresponding objective function and constraints are given below:

$$M_1\text{: } \min \sum_{t\in\mathcal{T}} \Big(\sum_{\omega\in\Omega^{NF}} p_\omega . \left(c^h I^+_{t,\omega} + c^b I^-_{t,\omega} \right) + \sum_{s\in\mathcal{S}} c^s Q^s_t \Big) \quad (1)$$

subject to constraints:

$$I^+_{t,\omega} - I^-_{t,\omega} = \sum_{s\in\mathcal{S}} \sum_{\tau=1}^{t-L^s_+} Q^s_\tau + \sum_{(s,\tau)\in\mathcal{M}^{NF}_t} \alpha^s_{\tau,\omega} Q^s_\tau - \sum_{\tau=1}^{t} D_\tau \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega^{NF} \quad (2)$$

$$Q^s_t = \sum_{\tau\in\mathcal{T}} y^s_{t,\tau} D_\tau \qquad \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \quad (3)$$

$$y^s_{\tau,t} \leq D_t \qquad \forall s \in \mathcal{S}, \forall t, \tau \in \mathcal{T} \quad (4)$$

$$\frac{D_t}{\sum_{i\in\mathcal{T}} D_i} \leq \sum_{s\in\mathcal{S}} \sum_{\tau=t-L^s_+}^{t-L^s_-} y^s_{\tau,t} \leq 1 \qquad \forall t \in \mathcal{T} \quad (5)$$

$$y^s_{\tau,t} \in \{0,1\} \qquad \forall s \in \mathcal{S}, \forall t, \tau \in \mathcal{T} \quad (6)$$

$$I^+_{t,\omega} \geq 0 \qquad \forall t \in \mathcal{T}, \forall \omega \in \Omega^{NF} \quad (7)$$

$$I^-_{t,\omega} \geq 0 \qquad \forall t \in \mathcal{T}, \forall \omega \in \Omega^{NF} \quad (8)$$

$$Q^s_t \geq 0 \qquad \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \quad (9)$$

The inventory level $I_{t,\omega}$ at the end of each period t for each scenario ω is expressed by Constraints (2). Since demands and ordered quantities are integers, then each inventory level is necessarily integer and can be positive if $I_{t,\omega} > 0$ (denoted by $I^+_{t,\omega}$) or negative if $I_{t,\omega} < 0$ (denoted by $I^-_{t,\omega}$). Dissatisfied demands will raise the backlog level of the period and remain in the system until they are satisfied if the inventory level is null at any period. Each demand cannot be dispatched between several periods or several suppliers. Thus, each non-zero ordered quantity Q^s_t is equal to the sum of one or several demands (Constraints (3)). We remind that $y^s_{\tau,t}$ is a binary variable (Constraints (6)). It is non-null if at period τ, we order D_t from supplier s. If D_t is null, no quantity is ordered from any supplier (Constraints (4)). If $D_t > 0$, at most one order is released (Constraints (5)). Constraints (7-8) define $I^+_{t,\omega}$ and $I^-_{t,\omega}$. These two values could be arbitrarily large, but they will not do so because in the objective function (1) they are associated to positive costs. Constraints (9) are non-negativity constraints. Knowing that all demands are integers, then Constraints (3) force Q^s_t to be integer variables.

B. Characterizing the scenarios of "With Flexibility" strategy

To account for the uncertainty of lead times and the flexibility, the following elements are introduced:

- the set of indices $\mathcal{M}^{WF}_t$ of all ordered quantities $Q^s_{\tau,i}$ used to calculate the backlogging level at t
- the set of aggregated scenarios Ω^{WF}
- the probability p_ω of each aggregated scenario $\omega \in \Omega^{WF}$

To calculate the inventory level at period t, we can split all possible $Q^s_{\tau,i}$ (each quantity is ordered at period τ from supplier s to satisfy demand i) into three groups: (i) orders that we are sure to receive at the latest at the end of period t ($\tau \leq t - L^s_+$), (ii) orders that will never be received before the end of period t ($\tau > t - L^s_-$), and (iii) orders that contain uncertainties about their arrival at period t ($t - L^s_+ + 1 \leq \tau \leq t - L^s_-$, see Fig. 2). Note that only the

order quantities belonging to the third group are subject to lead time uncertainty. Therefore, they are used to define the set $\mathcal{M}_t^{NF}$.

Definition III.3. *Let $\mathcal{M}_t^{WF}$ be the set of indices of all ordered quantities $Q^s_{\tau,i}$ that can be involved in the calculation of the backlogging level at period t. This set is used in the M_2 model, which considers the orders' flexibility. It is defined by:*

$$\mathcal{M}_t^{WF} = \left\{ (s,\tau,i) \in \mathcal{S} \times \mathcal{T}^2 : t - L^s_+ + 1 \leq \tau \leq t - L^s_- \right. \\ \left. \text{and } \tau + L^s_- \leq i \leq \tau + L^s_+ \right\}$$

The difference of $\mathcal{M}_t^{WF}$ compared to $\mathcal{M}_t^{NF}$ is that, by considering the flexibility on the orders, a quantity ordered from a supplier s at period τ is divided according to the demands to be satisfied. Then, each demand D_i can be satisfied entirely or by several portions delivered independently from different suppliers. Thus, $Q^s_{\tau,i}$ is precisely the quantity ordered from s, released at period τ to satisfy the demand of period i.

Each quantity $Q^s_{\tau,i}$ ordered at period τ from supplier s to satisfy the demand D_i arrives after a random lead time $L^s_{\tau,i,\omega}$. The periods in which this quantity can arrive are within the interval $[\tau+L^s_-, \tau+L^s_+]$. Therefore, the inventory level at any period τ depends on the realization of lead times of the ordered quantities indexed in the set $\mathcal{M}_t^{WF}$.

Definition III.4. *Let $\alpha^s_{\tau,i,\omega} = \mathbb{1}_{\{L^s_{\tau,i,\omega} \leq t-\tau\}} : (s,\tau,i) \in \mathcal{M}_t^{WF}$ be an indicator that expresses if the quantity ordered from supplier s at period τ to satisfy the demand D_i arrives before period t for a given scenario ω:*

$$\alpha^s_{\tau,i,\omega} = \begin{cases} 0 & \text{if } \tau + L^s_{\tau,i,\omega} > t, \text{ with } \mathbb{P}(L^s_{\tau,i,\omega} > t-\tau) \\ 1 & \text{if } \tau + L^s_{\tau,i,\omega} \leq t, \text{ with } \mathbb{P}(L^s_{\tau,i,\omega} \leq t-\tau) \end{cases}$$

The number of possible scenarios is equal to $|\Omega^{WF}| = 2^{|\mathcal{M}_t^{WF}|}$. It only depends on the cardinality of the set of triplets (s,τ,i). A given scenario ω is composed of a set of $\alpha^s_{\tau,i,\omega}$ for all $(s,\tau,i) \in \mathcal{M}_t^{WF}$. This allows to define the set of all possible aggregated scenarios as follows:

$$\Omega^{WF} = \left\{ \left(\alpha^s_{\tau,i,\omega}\right)_{(s,\tau,i)\in\mathcal{M}_t^{WF}} : w \in \{1,...,2^{|\mathcal{M}_t^{WF}|}\} \right\}$$

Property III.2. *Each scenario ω is characterized by its probability of occurrence p_w as defined below:*

$$p_\omega = \prod_{(s,\tau,i)\in\mathcal{M}_t^{WF}} \left(\begin{array}{l} \alpha^s_{\tau,i,\omega}\mathbb{P}(L^s_{\tau,i,\omega} \leq t-\tau) + \\ (1-\alpha^s_{\tau,i,\omega})\mathbb{P}(L^s_{\tau,i,\omega} > t-\tau) \end{array} \right) \forall \omega \in \Omega^{WF}$$

where $\alpha^s_{\tau,i,\omega} = \mathbb{1}_{\{L^s_{\tau,i,\omega} \leq t-\tau\}}$ is a binary value, and $\sum_{\omega\in\Omega^{WF}} p_\omega = 1$.

Based on the aggregation strategy resulting in the sets $\mathcal{M}_t^{WF}$ and Ω^{WF}, a second stochastic model, noted by $M2$, is proposed in the next sub-subsection. It is denoted by M_2 and allows both flexibility on orders and order splitting. This model is based on a scenario approach. The set of all possible scenarios $\omega \in \Omega^{WF}$ is considered to optimize the ETC. In this model formulation, it is assumed that both flexibility and splitting are allowed. This authorizes splitting each demand into several quantities that are ordered from different suppliers. Orders released from each supplier at a given period are separated and have independent lead times occurrences. This strategy can be formulated as the stochastic ILP given in Equations (10-16).

$$M_2\text{: } \min \sum_{t\in\mathcal{T}} \Big(\sum_{\omega\in\Omega^{WF}} p_\omega . \left(c^h I^+_{t,\omega} + c^b I^-_{t,\omega} \right) + \sum_{s\in\mathcal{S}} \sum_{\tau\in\mathcal{T}} c^s Q^s_{\tau,t} \Big) \tag{10}$$

subject to constraints:

$$I^+_{t,\omega} - I^-_{t,\omega} = \sum_{s\in\mathcal{S}} \sum_{\tau=1}^{t-L^s_+} \sum_{i=\tau+L^s_-}^{\tau+L^s_+} Q^s_{\tau,i} + \sum_{(s,\tau,i)\in\mathcal{M}_t^{WF}} \alpha^s_{\tau,i,\omega} Q^s_{\tau,i} - \sum_{\tau=1}^{t} D_\tau \qquad \forall t \in \mathcal{T}, \forall \omega \in \Omega^{WF} \tag{11}$$

$$Q^s_{\tau,t} \leq D_t \qquad \forall s \in \mathcal{S}, \forall t,\tau \in \mathcal{T} \tag{12}$$

$$\sum_{s\in\mathcal{S}} \sum_{\tau=t-L^s_-}^{t-L^s_+} Q^s_{\tau,t} = D_t \qquad \forall t \in \mathcal{T} \tag{13}$$

$$Q^s_{\tau,t} \in \mathbb{N} \qquad \forall s \in \mathcal{S}, \forall t,\tau \in \mathcal{T} \tag{14}$$

$$I^+_{t,\omega} \geq 0 \qquad \forall t \in \mathcal{T}, \forall \omega \in \Omega^{WF} \tag{15}$$

$$I^-_{t,\omega} \geq 0 \qquad \forall t \in \mathcal{T}, \forall \omega \in \Omega^{WF} \tag{16}$$

where $\alpha^s_{\tau,i,\omega}$ is the binary indicator introduced in Definition III.4, and p_ω is the probability of occurrence of scenario $\omega \in \Omega^{WF}$ given in Property III.2.

We examine all possible aggregated scenarios in the M_2 model, which is explained by Equations (10-16). We aim to minimize the ETC, including inventory, backlogging, and purchasing costs. We also need to determine the proportion of a given D_t that should be ordered from a given supplier s at a given period τ. The inventory level $I_{t,\omega}$ at the end of each period t for each scenario ω is expressed by constraints (11). Constraints (12) ensure that the quantity ordered from supplier s at period τ to satisfy the demand of period t is less than D_t. Constraints (13) guarantee the satisfaction of all demands and require the sum of the quantity ordered to satisfy the demand of period t equal to D_t.

Constraints (14) define the ordered quantities as integer variables. Constraints (15-16) define $I^+_{t,\omega}$ and $I^-_{t,\omega}$. These two values could not be arbitrarily large because in the objective function (10), they are associated with positive costs.

IV. Numerical Example

The two models were implemented in $C++$, solved by IBM CPLEX 12.9 and executed on a single core from a single node of a cluster. The used node is a 2 CPU Intel Xeon E5-2660 v3 with 10 cores at 2.60 GHz and 64 Go RAM. This small numerical example illustrates the difference between the four aforementioned strategies. The planning horizon has 8 periods ($\mathcal{T} = \{1,\ldots,8\}$) within which the periods 5 to 8 have a demand greater than zero, namely $D_5 = 30$, $D_6 = 23$, $D_7 = 10$ and $D_8 = 55$. The unit time backlogging and holding costs are $c^b = 15$ and $c^h = 10$, respectively. Supply can be ensured by 3 suppliers with different purchasing costs and lead time probability distributions, as presented in Table II.

TABLE II: Suppliers' data.

		l : lead time possible values				
s	PMF^s	1	2	3	4	p^s
1	$\mathbb{P}(L^s = l)$	0.24	0.76			68
2	$\mathbb{P}(L^s = l)$		0.53	0.16	0.31	65
3	$\mathbb{P}(L^s = l)$			0.48	0.52	65

PMF^s: Probability mass function of supplier's s lead time.

Tables III to IV detail the results of the optimal solutions for M_1, and M_2 models, respectively.

For these tables, the first row has the periods. The second displays the demands to be satisfied. The third shows the expected inventory value to be carried out during each period. Row 4 gives the expected value of the backlog level at each period. The fifth presents the expected arrivals. Finally, the quantities ordered from suppliers 1, 2, and 3 are shown in the last three.

Table III illustrates the optimal solution when neither orders splitting nor release flexibility are supported (cf. Section III-A). It

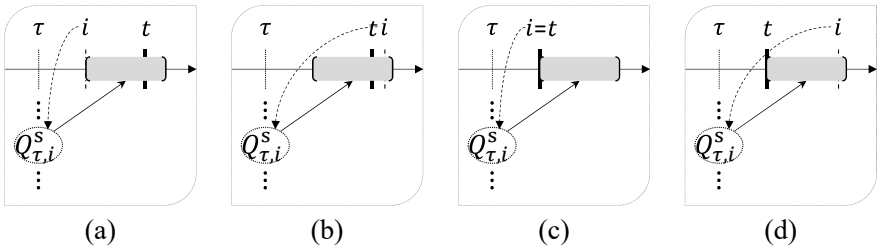

Fig. 2: Different cases representing the set $\mathcal{M}_t^{WF}$ for WF strategy.

can be observed that the optimal solution is at all the times ordered from the third supplier (30 units in period 1, 23 in period 2, 10 in period 3, and 55 in period 4). In other words, for all demands, we have a planned lead time equal to 4 periods (for example, demand D_5 is ordered at period 1). This solution presents the best compromise between inventory, backlogging, and purchasing costs. The lead time of supplier 3 is varying between 3 and 4 periods. So at period 4 we can only receive the quantity ordered at period 1 (Q_1^3=30 units) with the probability 0.48. The expected inventory level for all scenarios at this period equals 14.40 (30×0.48).

At period 5, the demand to satisfy is 30 units (D_5), and the quantity to be surely received before the end of this period is $Q_1^3 = 30$ units. However, there is uncertainty about the receipt of $Q_2^3 = 23$, which may arrive either at period 5 with a probability of 0.48 or at period 6 with a probability of 0.52. Therefore, the expected inventory to carry at the end of this period is $\mathbb{E}(I_5^+) = (30-30) \times 0.52 + (30+23-30) \times 0.48 = 11.04$. The optimal ETC is equal to 8236.4, which is the sum of expected inventory holding costs (566.4) and purchasing costs (7630).

Table IV entails the optimal solution where both flexibility and order splitting are considered (cf. Section III-B). The M_2 model proposes to order only the quantity $Q_{6,6}^1 = 10$ from the first supplier at period 6 and the rest from the third one at periods 1, 2, and 4. The optimal cost obtained by this model is 8119.26.

This numerical example illustrated the benefits of order splitting and their release flexibility in absorbing the effects of lead time uncertainty. More generally, these results also confirm that introducing flexibility reduces costs. The solution given by the M_2 model finds the best way to dispatch demands to different suppliers at different periods to get the best compromise between the cost of lead time uncertainty and purchasing cost. However, the stochastic M_2 model not only has much more scenarios than the M_1 model to be explored when evaluating the expected cost of a given solution, but also more decision variables, implying a wider space of solutions.

In this example, for M_1 model, for each scenario, it is necessary to fix $|\mathcal{M}_t^{NF}|$ values of $\alpha_{\tau,\omega}^s \forall (s,\tau) \in \mathcal{M}_t^{NF}$. By considering three suppliers, $|\mathcal{M}_t^{NF}| = \sum_{s\in\mathcal{S}} L_+^s - L_-^s = 4$. Thus, the number of all possible aggregated scenarios is $|\Omega^{NF}| = 2^4$.
For each scenario, the M_2 model must fix $|\mathcal{M}_t^{WF}|$ values of $\alpha_{\tau,i,\omega}^s$. By considering three suppliers, $|\mathcal{M}_t^{WF}| = \sum_{s\in\mathcal{S}} (L_+^s - L_-^s + 1) \times (L_+^s - L_-^s) = 2+3\times 2+2 = 10$. Thus, the number of all possible aggregated scenarios $|\Omega^{WF}|$ increases to 2^{10}.

Remark 1. *If one of the suppliers increases its uncertainty by one period, for M_2 model, $|\mathcal{M}_t^{WF}|$ will be equal to 14 and $|\Omega^{WF}|$ to 2^{14}. For M_1 model, $|\mathcal{M}_t^{NF}|$ will be equal to 5 and then $\Omega^{NF} = 2^5$.*

V. Conclusion

This paper presents a multi-period production planning and dynamic supplier selection problem with a known dynamic demand and stochastic lead times. For each supplier, order lead times are independent, discrete random variables with known and bounded probability distributions. Two linear stochastic models are presented to consider two alternative strategies relative to orders' splitting and ordering release flexibility.

The proposed models consider the order crossover and optimize the selection of suppliers (each supplier is characterized by a purchasing cost and a probability distribution for its lead time).

The results show that introducing flexibility in order delivery and the dispatching of demands between several suppliers is the best way to cope with the uncertainty of lead times.

Future work will focus on deeper analyses of the dependence between the suppliers' selection and the incurred costs due to uncertainty and purchasing costs. The objective is to identify effective strategies for negotiation between buyers and suppliers. It will also be interesting to add to our model the possibility of choosing between different transportation modes and include other criteria for supplier selection, such as quality and environmental impact.

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TABLE III: Solution of the numerical example using the M_1 model.

t	1	2	3	4	5	6	7	8
D_t					30	23	10	55
$\mathbb{E}(I_t^+)$				14.40	11.04	4.80	26.40	
$\mathbb{E}(I_t^-)$								
$\mathbb{E}(A_t)$				14.40	26.64	16.76	31.60	28.60
Q_t^1								
Q_t^2								
Q_t^3	30	23	10	55				

ETC = 8236.4

TABLE IV: Solution of the numerical example using the M_2 model.

t	1	2	3	4	5	6	7	8
D_t					30	23	10	55
$\mathbb{E}(I_t^+)$				3.36	5.30		20.86	
$\mathbb{E}(I_t^-)$					6.26		2.06	
$\mathbb{E}(A_t)$				3.36	25.72	23.92	28.80	36.20
$(Q_{t,5}^1, .., Q_{t,8}^1)$						$(0, 10, 0, 0)$		
$(Q_{t,5}^2, .., Q_{t,8}^2)$								
$(Q_{t,5}^3, .., Q_{t,8}^3)$	$(7, 0, 0, 0)$	$(23, 23, 0, 0)$		$(0, 0, 10, 45)$				

ETC = 8119.26

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