Practical extension of MACBETH methodology to deal with sophisticated preferences' models

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1st Abdelhak Imoussaten
EuroMov Digital Health in Motion, Univ Montpellier, IMT Mines Ales, Ales, France
0000-0002-1292-2681

***Abstract*—When solving complex and critical decision problems involving several aspects, collecting decision-maker (DM) preferences remains a delicate issue. Indirect parameter identification procedures are more adequate in such situations than procedures consisting to ask directly the DM to provide preferences. In the case of methods based on additive models such as MACBETH, disaggregation procedures are implemented to handle this problem. In case of sophisticated non-additive model, e.g. Choquet integral based fuzzy measure, most of the existing indirect procedures assume a 2-additive fuzzy measure and the DM is asked to change her/his preferences if they do not match this model. This paper proposes an approach to determine preferences model that achieves a trade-off between model sophistication and the fastidious MACBETH questioning procedure when considering interactions between more than two attributes. First, it proposes a practical approach to solve a real case of multiple criteria decision problem while considering sophistical model of preferences. On one hand, particular binary alternatives are selected for a first questioning procedure to determine the fuzzy measure parameters. In order to make the later procedure as simple as possible to handle, some simplifications are introduced to reduce both the amount of information and the cognitive effort requested from the DM. Our proposition consists in fixing several non-linear optimization problems until reaching the first k for which the k-additive fuzzy measure match the DM preferences. On the other hand, starting from a decision matrix and the answers to a second questioning procedure, value functions are determined. The second contribution is a theoretical one and consists in proving the existence of a k-additive fuzzy measure matching the DM preferences. Finally, the approach is applied to a real world case that concerns the comparison of some electrical and thermal vehicles regarding their environmental impacts.**

***Index Terms*—k-additive fuzzy measure, disaggregation procedure, Choquet integral, multiple criteria decision analysis**

I. Introduction

Let us consider the context of a multiple criteria decision problem where a decision maker (DM) needs support to resolve a rank decision problem. Let us assume that the DM agreed with the aggregation principle, i.e. poor performances of an alternatives on some criteria can be compensated by good performances on other criteria. Consequently, the decision problem is treated within the multi-attribute value theory (mavt) framework [1]. More precisely, let us consider that the alternatives are evaluated using a set of attributes $N = \{1, ..., n\}$ where the i^{th} attribute takes its values in a space X_i. An alternative is therefore an element $x = (x_1, ..., x_n)$ from the Cartesian product set $X = X_1 \times ... \times X_n$. In the mavt framework, the alternatives are compared using a value function $v : X \to [0, 1]$ that exists when the binary relation, denoted $\succsim$, representing the preferences of the DM over elements of X is a weak order. The most practical way to determine v is to find its analytical form using the partial value functions $v_i : X_i \to [0, 1]$, $i \in N$. The additive form of v requires that the attributes be mutually preference independent [2] which is a very restrictive condition. Indeed, this condition does not allow the expression of some kinds of preferences which take into account the interactions between attributes. A more general and flexible form is the decomposable model of Krantz *et al.* [3].

Definition 1: Under the hypotheses of 1) weak preference independence between attributes [2] and 2) $\succsim$ is a weak order, v has the following form [3]:

$$\forall x \in X, \quad v(x) = F(v_1(x_1), ..., v_n(x_n)) \tag{1}$$

where F is an aggregation operator increasing in all its arguments.

Note that the *weak preference independence* assumption simplifies the assessment of v by assessing v_i for a single attribute independently (the other attributes are fixed at arbitrarily selected values) [2]. An interesting candidate for the operator F is the Choquet integral operator [4] related to a fuzzy measure [5]. The fuzzy measure can model interactions between attributes and when these interactions exist, the function v of (1) is non-additive. For such sophisticated models it is difficult to obtain the parameters information from the DM, in particular, when considering interaction for more than two attributes. For instance, the authors of [6] [7] point out the discrepancy that might exist between the meaning of the values assigned to the interactions and the preferences in the mind of the DM. This led several works to consider Choquet integral based 2-additive fuzzy measure models where the interactions are limited to two attributes. These models are a trade-off between an additive model and a sophisticated non-linear one. Indeed, with a 2-additive fuzzy measure there are fewer parameters and the DM can understand their meaning. To obtain the coefficients of a 2-additivity fuzzy measure, disaggregation procedures are preferred to asking the DM directly to provide these parameters, as they have the advantage of requiring less cognitive effort from DM [8] [9] [10]. MACBETH [11] is a Multiple Criteria Decision Analysis (MCDA) method that offers an example of a well established disaggregation procedure for an additive model. Several extensions to non-additive models were proposed [9] [12] [13]. However, in case of 2-additivity model in some situations the information provided by the DM fails to provide solutions. Then some approaches attempt to find the parameters that minimize the error with the DM preferences and some others propose to ask the DM to modify the information provided until they are coherent with a 2-additive fuzzy measure based model. This paper proposes a practical extension of MACBETH method to deal with k-additive model while attempting to minimize the cognitive effort of the DM and shows how this extension can be used in a real world problem. The theoretical aspects of the approach are close to the proposition in [9] concerning the general extension of MACBETH to non-additive models and [12] [13] concerning the condition of the existence of 2-additive model corresponding to DM preferences, while it stands out by proposing 1) several optimization programs dedicated to determine the k-additive model with the smallest k in case of interactions between more than two attributes; 2) particular learning examples for which it is easy for the DM to provide preferences and give her/him the possibility of expressing interactions between attributes; 3) a proof for the existence of a k-additive model coherent with these preferences. The paper is organised as follows. Section II gives the theoretical background of the paper and the reminder about the MACBETH questioning procedure. We present our approach in Section III and provide an illustration concerning the comparison of some electrical and thermal vehicles regarding their environmental impacts in Section IV.

II. Theoretical background and Reminders

To simplify, we adopt in the rest of the paper the following notations for $i, j \in N$, $i \neq j$, and $I \subseteq N$: the subset $\{i\}$ denoted by i, the subset $N \setminus \{i\}$ denoted by $-i$, the subset $\{i, j\}$ by ij, the subset $N \setminus \{i, j\}$ denoted by $-ij$ and more general, the subset $N \setminus I$ denoted by $-I$. Similarly, for some vectors of X (rep. $\mathbb{R}^n_+$), for $x, y \in X$ (rep. $\mathbb{R}^n_+$), (x_I, y_{-I}) denotes the vector $z \in X$(rep. $\mathbb{R}^n_+$) such that $z_i = x_i$, $\forall i \in I$ and $z_i = y_i$, $\forall i \in N \setminus I$.

A. Fuzzy Measure and Möbius Transform

Definition 2: A *fuzzy measure* μ over N is a set functions from 2^N to $[0, 1]$ verifying [5]:

1) boundary conditions: $\mu(\emptyset) = 0$ and $\mu(N) = 1$;
2) monotonicity conditions: $\forall K, T \subseteq N$,

$$K \subseteq T \Longrightarrow \mu(K) \leq \mu(T).$$

Remark 1: Let us consider a subset $I \subseteq N$, the quantity $\mu(I)$ represents the weight of the coalition I of attributes in the decision [4] and the monotonicity of μ means that the weight of I can not decrease when adding to it an other attribute.

The definition 2 does not allow to find $\mu(K)$ from $\mu(T)$, and vice versa $\forall K, T \subseteq N$. Thus, in order to determine μ one has to ask $2^n - 2$ parameters from the DM. To reduce the number of parameters defining a fuzzy measure, the notion of Möbius transform was introduced in [14] [15].

Definition 3: The Möbius transform of a fuzzy measure μ over N is a function $m_\mu : 2^N \rightarrow [0, 1]$ defined as:

$$m_\mu(T) = \sum_{K \subseteq N : K \subseteq T} (-1)^{|T \setminus K|} \mu(K), \quad \forall T \subseteq N \qquad (2)$$

When m_μ is given, it is possible to retrieve μ:

$$\mu(T) = \sum_{K \subseteq N : K \subseteq T} m_\mu(K), \quad \forall T \subseteq N \qquad (3)$$

In order that coefficients $m_\mu(T), T \subseteq N$ correspond to a Möbius transform of a fuzzy measure μ over N, the boundary and monotonicity conditions of the fuzzy measure must be ensured [14]:

1) boundary conditions:

$$m_\mu(\emptyset) = 0 \ and \ \sum_{T \subseteq N} m_\mu(T) = 1;$$

2) monotonicity conditions: $\forall T \subseteq N$ and $\forall i \in N$,

$$\sum_{K : \{i\} \subseteq K \subseteq T} m_\mu(K) \geq 0.$$

The advantage of introducing a Möbius transform is that under certain conditions, one can define a fuzzy measure with a very small number of parameters. Indeed the notion of k-additive fuzzy measure [16] consider a reasonable number of interactive attributes in order to achieve a trade-off between the quantity of information required from the DM to define the fuzzy measure parameters and the sophisticated model of v in (1).

Definition 4: Let $k \in N$. A fuzzy measure μ is k-additive if its Möbius transform m_μ verify:

1) $m_\mu(T) = 0$, $\forall T \subseteq N$ such that $|T| > k$;
2) $\exists K \subseteq N$ such that $|K| = k$ and $m_\mu(K) \neq 0$.

From Definition 4, one can deduce that a k-additive fuzzy measure is completely defined by the coefficients $m_\mu(T)$, $\forall T \subseteq N : |T| \leq k$. This means that only $\sum_{l=1}^{k} \binom{n}{l}$, e.g. $\frac{n(n+1)}{2}$ in the case when $k = 2$, coefficients are needed to build a k-additive fuzzy measure instead of $2^n - 2$. Thus for a k-additive fuzzy measure, we have:

$$\mu(I) = \sum_{\emptyset \neq T \subseteq I, \ |T| \leq k} m_\mu(T), \qquad \forall I \subseteq N. \qquad (4)$$

To facilitate the understanding of the fuzzy measure coefficients, researchers have tried to express them using several quantities as *Shapley* values [17] and interactions quantification [18] [16]:

Definition 5: The *Shapley* value express a power index for an attribute $i \in N$:

$$\nu_i = \sum_{I \subseteq N \setminus i} \frac{(n - |I| - 1)!|I|!}{n!} [\mu(I \cup i) - \mu(I)] \qquad (5)$$

ν_i can be interpreted as a weighted average value of the marginal contribution of attribute i alone in all coalitions [19].

Definition 6: The interaction between a pair of attributes $i, j \in N$ can be expressed as:

$$I_{ij} = \sum_{I \subseteq N \setminus ij} \frac{m(I \cup ij)}{|I| + 1} \tag{6}$$

I_{ij} can be interpreted as a weighted average value of the added value produced by putting i and j together, all coalitions being considered [19]. Depending on the sign of I_{ij}, attributes can have three kind of synergies [19]: 1) positive synergy: $I_{ij} > 0$ (complementarity); 2) negative synergy: $I_{ij} < 0$ (redundancy); and 3) no synergy: $I_{ij} = 0$ (the attributes are independents).

B. Choquet Integral

Definition 7: In a MCDA context, the Choquet integral of a vector of positive real numbers $y = (y_1, ..., y_n) \in \mathbb{R}^n_+$ *w.r.t* a fuzzy measure μ, denoted $C_\mu(y)$, is defined as:

$$C_\mu(y) = \sum_{i=1,n} (y_{\sigma(i)} - y_{\sigma(i-1)})\mu(A_\sigma(i)) \tag{7}$$

where $0 = y_{\sigma(0)} \leq y_{\sigma(1)} \leq y_{\sigma(2)} \leq ... \leq y_{\sigma(n)}$ (σ is a permutation over N) and $A_\sigma(i) = \{\sigma(i), \sigma(i+1), ..., \sigma(n)\}$. Equation (7) can be expressed using the Möbius transform m_μ as follows:

$$C_\mu(y) = \sum_{I \subseteq N} m_\mu(I) \min_{i \in I} y_i \tag{8}$$

Example 1: Let us consider particular vectors from $\mathbb{R}^n_+$: $(1_i, 0_{-i})$, $(1_{ij}, 0_{-ij})$ where $i, j \in N, i \neq j$. We can easily calculate the Choquet integral of these vectors using (7) and (8):

$$C_\mu(1_i, 0_{-i}) = \mu(i) = m(i)$$

$$C_\mu(1_{ij}, 0_{-ij}) = \mu(ij) = \sum_{I \subseteq ij} m(I) = m(i) + m(j) + m(ij).$$

C. Disaggregation Procedure

Except some very specific cases, asking directly v_i's and Möbius's values requires a considerable cognitive effort from the DM especially when considering interactive attributes. For this reason disaggregation procedure is introduced in the MCDA works in order to identify the methods parameters. The main idea is to ask the DM about information which is for him easy to provide and then from this information v_i, $i \in N$ and $m(I)$, $I \subseteq N$ are inferred. The corresponding information concern the preferences of the decision-maker about some alternatives that are easy for him to compare. The principles of the disaggregation procedure in MCDA methods is the same as the one used in the machine learning techniques. For example, in the supervised classification task, for some classifiers a model is assumed to separate classes then the parameters of this model are determined by restoring the model from learning examples. In this paper we consider the disaggregation procedure of MACBETH method [20]. The advantages of this later procedure, proposed initially for an additive model, are numerous. First it proposes a questioning procedure from which the preferences are collected and the selected alternatives are easy to compare by the DM. Second, MACBETH method states a linear program where the constraints consist of the DM preferences and the decision variable are the model parameters. The details of the questioning procedure are presented in Subsubsection II-C1 and the details of the established linear program are presented in Subsubsection II-C2.

1) MACBETH Questioning Procedure: The MACBETH questioning procedure is organised as follows:

- Q1: Is one of the two elements more attractive than the other? DM's response (R1) can be: "Yes", or "No", or "I don't know". If R1 = "Yes", a second question (Q2) is asked:
- Q2: Which of the two elements is the most attractive?
- Q3: How do you judge the difference of attractiveness?
- DM's response (R3) would be provided in the form "d_s", where d_s, $s = 1, 2, .., 6$ are semantic categories of difference of attractiveness defined so that, if $s < s'$, the difference of attractiveness d_s is weaker than the difference of attractiveness $d_{s'}$. The Six semantic categories of difference of attractiveness are "**very weak**", "**weak**", "**moderate**", "**strong**", "**very strong**" or "**extreme**".

In the MACBETH methodology, it is assumed that the DM is able to identify for each attribute $i \in N$ two absolute reference levels:

- A first reference level denoted $\mathbf{1}_i$ in X_i which it is considered good and completely satisfying for the DM, even if more attractive elements could exist: $v_i(\mathbf{1}_i) = 1$.
- A second reference level denoted $\mathbf{0}_i$ in X_i which is considered completely unacceptable by the DM: $v_i(\mathbf{0}_i) = 0$.

We denote $\mathbf{0} \in X$ (resp. $\mathbf{1} \in X$) the n-dimensional vector where all the components are $\mathbf{0}_i$ (resp. $\mathbf{1}_i$). These two reference levels are necessary to ensure commensurateness between attributes. Two parts are involved in the MACBETH disaggregation procedure for a model as in (1) when F is a weighted average operator. The first one consists in determining the weights ω_i and the second one consists in determining the value functions v_i for each attributes (n times). For the ith attribute, to determine v_i the DM compares the fictive alternatives $(x_i^j, \mathbf{0}_{-i})$ where x^j correspond to the jth alternatives that the DM aims to rank. While to determine ω_i, the DM is asked to compare binary alternatives $(\mathbf{1}_i, \mathbf{0}_{-i})$. Note that the reference alternatives $\mathbf{1}$ and $\mathbf{0}$ are also included in the comparisons in the two steps. As one can remark these alternatives are easy to compare and they give information about the unknown parameters. In addition, in case of a small number of alternatives (resp. attributes), it is not fastidious to compare alternatives $(x_i^j, \mathbf{0}_{-i})$ (resp. $(\mathbf{1}_i, \mathbf{0}_{-i})$). In the case of l alternatives to rank, the DM has to handle $l\ (l+1)/2$ pair comparisons n times for determining all v_i. While for determining ω_i, the DM has $n\ (n+1)/2$ comparisons to handle. Based on the answers, MACBETH detects any inconsistencies and offers corrections interactively. Three binary preference relations can be distinguished from the answers.

- $\sim = \{(a, b) \in \mathbb{A} \times \mathbb{A} : a$ is not more attractive than b and b is not more attractive than $a\}$;

- $\succ = \{(a, b) \in \mathbb{A} \times \mathbb{A} : a$ is more attractive than $b\}$; and when the DM provides qualitative judgements about the difference of attractiveness, the following binary relation is considered:

$$P_s = \{a \succ b \text{ and the difference of attractiveness is } d_s \}$$

where $\mathbb{A}$ is the set of compared alternatives, $s \in \{1, ..., 6\}$, $\succ$ is the asymmetric part of the binary relation $\succsim$ and $\sim$ its symmetric part. Note that introducing the difference of attractiveness when comparing alternatives lead to assume the condition of difference independence of preferences [2]. Finally, the ordinal information obtained from the decision-maker are: $\{\sim, P_s\}$. When considering a non-additive model, the questioning procedure for the step concerning the determination of v_i is the same as in the additive one. However, for practical reasons, alternatives $(x_i^j, \mathbf{1}_{-i}))$ are preferred to alternative $(x_i^j, \mathbf{0}_{-i}))$. Indeed, the DM could consider, in some situations, that all alternatives $(x_i^j, \mathbf{0}_{-i}))$ are equivalent to the alternative $\mathbf{0}$. While the extension of the questioning procedure when determining ω_i is not obvious. Indeed, the straightforward extension leads to ask the DM to make (2^n) $(2^n - 1)/2)$ comparisons for the binary alternatives $(\mathbf{1}_I, \mathbf{0}_{-I})$, $I \subseteq N$. In addition to the large number issue, some alternatives are difficult to compare. In [12] [13], the authors propose to focus on determining a 2-additive fuzzy measure by considering only the binary alternative with a maximum of two different attributes i and j fixed at levels $\mathbf{1}_i$ and $\mathbf{1}_j$. As mentioned before, in this case the issue is that in some situation a Choquet integral based 2-additive fuzzy measure does not match the DM preferences.

2) MACBETH Linear Program: MACBETH establishes a linear program where the ordinal preference information $\{\sim, P_s\}$ form a part of the constraints and the decision variables are the values of the interval scale v, *i.e.,* which is defined up to a positive linear transformation: $v \to \alpha v + \beta$, $\alpha > 0$ (slope) and $\beta \in \mathbb{R}$ (constant) [21] [19] with $v(\mathbf{0}) = 0$ and $v(\mathbf{1}) = 1$. In the case where the ordinal information concern alternatives $(x_i^j, \mathbf{0}_{-i})$, we have $v(x_i^j, \mathbf{0}_{-i}) = \omega_i \; v_i(x_i^j)$ and in the case of alternatives $(\mathbf{1}_i, \mathbf{0}_{-i})$, we have $v(\mathbf{1}_i, \mathbf{0}_{-i}) = \omega_i$. Let us denote by $y_1, \ldots, y_l$ the value function, i.e., the interval scale, of the l compared alternatives. The corresponding linear program is presented in Tab. I.

TABLE I
LINEAR PROGRAM TO DETERMINE VALUE FUNCTION.

min :	
subject to	
$y_p - y_r = 0$	$\forall (x^p, x^r) \in \sim$
$\sigma_{s-1} < y_p - y_r$	$\forall (x^p, x^r) \in P_s$
$y_p - y_r < \sigma_s$	$\forall (x^p, x^r) \in P_s$
$0 = \sigma_0 < \sigma_1$	
$\sigma_s < \sigma_{s+1}$	$s = 1, \ldots, 5$
$0 \leq y_j \leq 1$	$j = 1, \ldots, l$

III. MACBETH disaggregation procedure Extension

In this section we present our proposition for the MACBETH methodology extension to non-additive model. First, Subsection III-A presents the adaptation of the MACBETH questioning procedure related to the determination of attributes' weights to the determination of the Möbius transform coefficients. In this procedure, particular alternatives are presented to the DM in order to express preference related to the attribute pair interactions. Then we demonstrate that it exists a Choquet integral based fuzzy measure representing the collected preferences. In Subsection III-B, we present the extension of the linear program in the MACBETH method to a non-linear one that guaranties an uniform distribution of the coefficient in case where information about difference of attractiveness are not provided. Therefore, a mean square error is fixed as objective function. Finally, in Subsection III-C we show that it's not always guaranteed to obtain a 2-additive fuzzy measure and we propose an iterative procedure to determine the optimal k, i.e. the smallest one, such that a k-additive fuzzy measure correspond to the DM preferences.

A. Questioning Procedure

As mentioned in Subsection II-C1 the large number of alternatives to compare and the difficulties of comparisons for the DM require the adaptation of the questioning procedure of MACBETH methodology when a non-additive model is involved. The extension proposed in this paper consists in reducing the number of binary alternatives and the number of comparisons. As the aim is to consider as few as possible the interactions between attributes in order to facilitate the DM understanding of the model parameters, we propose to limit the binary alternatives to those highlighting interactions for maximum of two attributes. Those binary alternatives are $\{(\mathbf{0}_i, \mathbf{1}_{-i}), (\mathbf{0}_{ij}, \mathbf{1}_{-ij}), i \neq j, i, j \in N\}$.

The proposed questioning procedure is the following:

- First, the DM is asked to rank the binary alternatives from the best to the worst. The alternatives $\mathbf{1}$ is placed at the top of the ranking and the alternative $\mathbf{0}$ at the bottom of the ranking.
- Second, the DM is asked to provide a difference of attractiveness between each pair of successive alternatives in the ranking.

The advantage of this simplification of the MACBETH original procedure, where only the matrix cells located directly above the diagonal are considered, is twofold. First, the number of comparisons is drastically reduced and second the preference inconsistencies are limited. More precisely, let us denote $\mathbb{B} = \{\mathbf{0}, \mathbf{1}, \{(\mathbf{0}_i, \mathbf{1}_{-i}), (\mathbf{0}_{ij}, \mathbf{1}_{-ij}), i \neq j, i, j \in N\}\}$, the inconsistencies that can exist in the DM preferences when ranking the alternatives in $\mathbb{B}$ are encountered when the DM considers that $(\mathbf{0}_{ij}, \mathbf{1}_{-ij}) \succ (\mathbf{0}_i, \mathbf{1}_{-i})$. They are considered inconsistencies because they are in a contradiction with the properties of an aggregation operator and the DM can reasonably avoid them. If these kinds of inconsistencies are not present in the preferences then it exists a Choquet integral based fuzzy measure μ that represents the DM preferences.

Definition 8: The preferences $\{\sim, \{P_s, s \in \{1,\ldots,6\}\}\}$ over $\mathbb{B}$ are said to be consistent if

- $\forall i,j \in N$ such that $i \neq j$ then either $(\mathbf{0}_i, \mathbf{1}_{-i}) \succ (\mathbf{0}_{ij}, \mathbf{1}_{-ij})$ or $(\mathbf{0}_i, \mathbf{1}_{-i}) \sim (\mathbf{0}_{ij}, \mathbf{1}_{-ij})$,
- $\mathbf{1} \succ \mathbf{0}$.

The second condition cannot be satisfied when the DM considers that all the alternatives are totally good or totally bad. The drawback of the small comparisons related to the number of the alternatives is that the obtained value functions are not as precise as those that one would obtain with more comparisons.

Proposition 1: If the preferences

$$\{\sim, \{P_s, s \in \{1,\ldots,6\}\}\}$$

over $\mathbb{B}$ are consistent then it exists a Choquet integral based a fuzzy measure μ that represents the DM preferences.

Proof 1: The proof can be divided into two parts. The first part consists in proving that for a list of ranked object it is always possible to associate real values between 0 and 1, i.e., interval scale, respecting some constraints on the width of differences between successive ordered values, i.e., some widths are more larger then others. This can be done using mathematical induction and by considering 3, 4, ... elements in the ranked list. When adding a new element, one has just to adjust the other widths if necessary. The second part is dedicated to prove the existence of a Choquet integral based fuzzy measure μ that represents the preferences $\{\sim, \{P_s, s \in \{1,\ldots,6\}\}\}$ over $\mathbb{B}$ when the coefficient

$$\{\mu(-i), \mu(-ij), i \neq j, i,j \in N\}$$

are ordered values in $[0,1]$ such that $\mu(-i) \geq \mu(-ij)$, $\forall i,j \in N$ and $i \neq j$. In order to prove the second part, let us, for example, consider that $\mu(N) = 1$, $\mu(\emptyset) = 0$ and for each subset $I \subset N$ such that it exists at least a pair $i', j' \in N$ such that $i' \neq j'$ and $I \subset N \setminus \{i', j'\}$, we set

$$\mu(I) = \min_{i,j \in N, i \neq j, I \subseteq N \setminus \{i,j\}} \mu(-ij). \tag{9}$$

Then μ is fully defined and it is a fuzzy measure over N. Indeed, let us consider $A, B \subseteq N$ such that $A \subseteq B$ and let us prove that $\mu(A) \leq \mu(B)$. If $A = N \setminus i$, $i \in N$ then either $B = N \setminus i$ or $B = N$ thus in both cases $\mu(A) \leq \mu(B)$. Else if $A = N \setminus \{i,j\}$, $i,j \in N$ and $i \neq j$ then either $B = A$, or $B = N \setminus i$ or $B = N$ thus in all cases $\mu(A) \leq \mu(B)$. Elsewhere, it exists at least a pair $i', j' \in N$ such that $i' \neq j'$ and $A \subset N \setminus \{i', j'\}$. If it exists a pair $i'', j'' \in N$ such that $i'' \neq j''$ and $B = N \setminus \{i'', j''\}$ (or it exist $i'' \in N$ such that $B = N \setminus \{i''\}$) then $\mu(A) \leq \mu(B)$. If else we have from (9) $\mu(A) = \mu(B)$.

B. Optimization Program

In Subsection III-A, we show that it exists a fuzzy measure corresponding to the DM preferences. In this subsection we set a non-linear optimization program (see Tab. II) to determine the coefficients of this fuzzy measure. For presentation simplifications purpose, we denote the alternative of $\mathbb{B}$ in the ascending order corresponding to the ranking provided by the DM as $\{b^1, \ldots, b^p\}$ where $p = \frac{n(n+1)}{2} + 2$.

TABLE II
QUADRATIC OPTIMIZATION PROGRAM TO DETERMINE COEFFICIENTS.

min :	$\sum_{j \in \{1,\ldots,p\}} (v(b^j) - \frac{j}{p-1})^2$
subject to	
preferences constraints	
$y_t - y_r = 0$	$\forall (b^t, b^r) \in \sim, t, r \in \{1,\ldots,p\}$
$\sigma_{s-1} < y_t - y_r$	$\forall (b^t, b^r) \in P_s, t, r \in \{1,\ldots,p\}$
$y_p - y_r < \sigma_s$	$\forall (b^t, b^r) \in P_s, t, r \in \{1,\ldots,p\}$
$y_t = a$	$b^t = \mathbf{a}$, $a \in \{0,1\}$
$y_t = v(\mathbf{0}_i, \mathbf{1}_{-i}) = \sum_{K \subseteq N \setminus \{i\}} m(K)$	for $b^t = (\mathbf{0}_i, \mathbf{1}_{-i})$
$y_t = v(\mathbf{0}_{ij}, \mathbf{1}_{-ij}) = \sum_{K \subseteq N \setminus \{i,j\}} m(K)$	for $b^t = (\mathbf{0}_{ij}, \mathbf{1}_{-ij})$
$0 \leq y_t \leq 1$	$j = 1, \ldots, p$
fuzzy measure constraints	
k-additivity constraints:	
$m(T) = 0$	$\forall T \subseteq N : \lvert T \rvert > k$
boundary constraint:	
$\sum_{T \subseteq N} m(T) = 1$	
monotonicity constraints:	
$\sum_{K : \{i\} \subseteq K \subseteq T} m(K) \geq 0$	$\forall T \subseteq N$ and $\forall i \in N$
semantic categories of difference of attractiveness constraints	
$0 = \sigma_0 < \sigma_1$	
$\sigma_s < \sigma_{s+1}$	$s = 1, \ldots, 5$

As one can see in Tab. II, the objective function part of the optimization program is changed comparing to the original MACBETH linear program. The remaining parts simply translate the expression of the value function v as the Choquet integral related to a Möbius transform and the constraints related to the monotonicity of a fuzzy measure. Regarding the objective function, the proposed one is a mean square error between the values associated to the alternatives obtained from the preferences and an uniform distribution of $\frac{n(n+1)}{2}$ values between 0 and 1. The aim of this objective function is twofold. First, in case of no difference of attractiveness are provided by the DM, the associated values will correspond to an uniform distribution which corresponds to the well-known principle in artificial intelligence that is ”the least commitment principle”. Second, this allow us to avoid local solutions from the solver that are close to the boundaries of the decision variables.

C. Iterative Determination of a k-additive Fuzzy Measure

The existence of a Choquet integral based fuzzy measure representing the preferences $\{\sim, \{P_s, s \in \{1,\ldots,6\}\}\}$ do not guaranty that this fuzzy measure is a 2-additive one. Example 2 shows, for the case of 3 attributes, an example of preferences for which a Choquet integral based 2-additive fuzzy measure is not compatible.

Example 2: Let us consider the following example with 3 attributes and suppose that the DM provides the preferences in (10).

$$\begin{aligned}&(\mathbf{1},\mathbf{1},\mathbf{1}) \succ^1 (\mathbf{1},\mathbf{1},\mathbf{0}) \succ^3 (\mathbf{1},\mathbf{0},\mathbf{1}) \succ^4 (\mathbf{1},\mathbf{0},\mathbf{0})\\ &\sim (\mathbf{0},\mathbf{1},\mathbf{1}) \succ^4 (\mathbf{0},\mathbf{1},\mathbf{0}) \succ^1 (\mathbf{0},\mathbf{0},\mathbf{1}) \succ^1 (\mathbf{0},\mathbf{0},\mathbf{0})\end{aligned} \tag{10}$$

where for two alternatives $a, b \in X$, and a difference of attractiveness d_s, $a \succ^s b$ means a is preferred to b and the difference of attractiveness is d_s. If it exists a 2-additive fuzzy measure for which the Choquet integral corresponds to the preferences of (10), Equations (11) - (21) show the constraints that the corresponding möbuis coefficients should obey.

$(\mathbf{1},\mathbf{1},\mathbf{1}) \succ^1 (\mathbf{1},\mathbf{1},\mathbf{0})$:

$$\sigma_0 < 1 - (m_1 + m_2 + m_{12}) < \sigma_1 \tag{11}$$

$(\mathbf{1},\mathbf{1},\mathbf{0}) \succ^3 (\mathbf{1},\mathbf{0},\mathbf{1})$:

$$\sigma_2 < m_2 + m_{12} - m_3 - m_{13} < \sigma_3 \tag{12}$$

$(\mathbf{1},\mathbf{0},\mathbf{1}) \succ^4 (\mathbf{1},\mathbf{0},\mathbf{0})$:

$$\sigma_3 < m_3 + m_{13} < \sigma_4 \tag{13}$$

$(\mathbf{1},\mathbf{0},\mathbf{0}) \sim (\mathbf{0},\mathbf{1},\mathbf{1})$:

$$m_1 = m_2 + m_3 + m_{23} \tag{14}$$

$(\mathbf{0},\mathbf{1},\mathbf{1}) \succ^4 (\mathbf{0},\mathbf{1},\mathbf{0})$:

$$\sigma_3 < m_3 + m_{23} < \sigma_4 \tag{15}$$

$(\mathbf{1},\mathbf{0},\mathbf{0}) \succ^4 (\mathbf{0},\mathbf{1},\mathbf{0})$:

$$\sigma_3 < m_1 - m_2 < \sigma_4 \tag{16}$$

$(\mathbf{0},\mathbf{1},\mathbf{0}) \succ^1 (\mathbf{0},\mathbf{0},\mathbf{1})$:

$$\sigma_0 < m_2 - m_3 < \sigma_1 \tag{17}$$

$(\mathbf{0},\mathbf{0},\mathbf{1}) \succ^1 (\mathbf{0},\mathbf{0},\mathbf{0})$:

$$\sigma_0 < m_3 < \sigma_1 \tag{18}$$

boundary constraint:

$$m_1 + m_2 + m_3 + m_{12} + m_{13} + m_{23} + m_{123} = 1 \tag{19}$$

2-additivity:

$$m_{123} = 0 \tag{20}$$

monotonicity:

$$\begin{aligned}&m_1 \geq 0, m_2 \geq 0, m_3 \geq 0,\\ &m_1 + m_{12} \geq 0,\\ &m_2 + m_{12} \geq 0,\\ &m_3 + m_{13} \geq 0,\\ &m_1 + m_{12} + m_{13} + m_{123} \geq 0,\\ &m_2 + m_{12} + m_{23} + m_{123} \geq 0,\\ &m_3 + m_{13} + m_{23} + m_{123} \geq 0.\end{aligned}$$

attractiveness constraints:

$$0 < \sigma_0 < \sigma_1 < \ldots < \sigma_6 \tag{21}$$

From (11) and (19) and (20), we have:

$$\sigma_0 < m_3 + m_{13} + m_{23} < \sigma_1 \tag{22}$$

From (13) and (18), we have:

$$m_{13} > \sigma_3 - \sigma_1 \tag{23}$$

From (15) and (18), we have:

$$m_{23} > \sigma_3 - \sigma_1 \tag{24}$$

Put together (21), (23) and (24) are in contradiction with (22). In case it does not exist a 2-additive fuzzy measure, instead of asking the DM to change the preferences, the paper proposes to identify the k-additive fuzzy measure where k is as small as possible regarding the preferences. For this aim an iterative algorithm (see Algorithm 1) is proposed that calls the quadratic program $solveQOP(l, \{\sim, \{P_s, s \in \{1, \ldots, 6\}\}\})$ of Tab. II for 1-additive, 2-additive, 3-additive, ..., until a solution is found.

Algorithm 1: Determining the k-additive fuzzy measure from preferences information.

```
Data: Preference structure {~, {P_s, s ∈ {1, ..., 6}}}
Result: A k-additive fuzzy measure μ
// initialisation
l ← 1;
repeat
    // solve the quadratic optimization problem for
       l-additive fuzzy measure
    if it exists a l-additive solution then
        μ ← solveQOP(l, {~, {P_s, s ∈ {1, ..., 6}}});
        k ← l;
    else
        l ← l + 1;
until it exists a l-additive solution μ
return μ;
```

IV. Illustration

To illustrate the proposed extension of MACBETH methodology, we consider an application involving a multiple criteria decision problem. The application concerns the comparison of five vehicles, three electrical ones and two thermal ones, based on their environmental impacts related to the life cycle assessment (LCA) principles. The application derives from two studies. The first one conducted by the french agency for ecological transition [22] dedicated to the LCA study and the second one [23] aimed to compare the vehicles using MCDA based on the decision matrix obtained with the first study and the preferences of some decision-makers from the french industry. Note that, only a subset of selected environmental impacts and a subset of selected alternatives are considered in the second study as the aim of the study was to show how one can use MCDA methods for decision making within LCA. Tab. III presents the list of the attributes and the alternatives considered in the second study and Tab. IV presents the corresponding decision matrix. Due to lack of space, we cannot give more details about the application (see [23] and [22] for more details). Note that the values in the decision matrix of Tab. IV represented the environmental impacts and therefore they have to be minimized.

TABLE III
ATTRIBUTES AND ALTERNATIVES.

attribute	number	abbreviation	alternative	abbreviation
Climate change	1	CC	Electric vehicle France	EV FR
Acidification	2	AC	Electric vehicle Germany	EV DE
Eutrophication	3	Eutro	Electric vehicle Europe (EU 27)	EV EU27
Total energy consumption	4	TEC	Gasoline thermal vehicle	Gas
Radioactive waste	5	RW	Diesel thermal vehicle	Dies
Radioactive emissions (air)	6	RE		
NOx emissions	7	Nox		

TABLE IV
THE DECISION MATRIX.

Attribute	EV DE	EV EU27	EV FR	Gas	Dies
CC (10^4 kg CO2 equiv)	1.78	1.49	0.678	2.69	2.22
AC (kg SO2 equiv)	47.8	70.3	34.3	41.5	49
Eurto (kg Phosphate equiv)	4	4.27	2.56	3.75	6.46
TEC (10^5 MJ)	3.09	2.99	3.02	4.11	3.32
RW (10^{-2} kg)	5.08	6.25	15.1	1.27	1.28
RE (Bq I129 equiv)	10.6	12.8	23.2	7.94	7.85
Nox (kg)	23.4	26.6	14.1	20	34.8

A. *Möbius Coefficients Determination*

We consider in this study the preferences of the DM that is a single member from the french industry. The question procedure in Subsection III-A is answered by the DM and provide preferences about the comparisons of binary alternatives in Tab. V. Note that we designed a very user-friendly questionnaire, that is not presented here for space reasons, to facilitate the DM task. In Tab. V are presented the DM's answers to the questionnaire.

TABLE V
THE DM PREFERENCES CONCERNING BINARY ALTERNATIVES.

$$\begin{array}{l}
(\mathbf{1,1,1,1,1,1,1}) \succ^3 (\mathbf{1,1,0,1,1,1,1}) \sim (\mathbf{1,0,1,1,1,1,1}) \sim (\mathbf{1,0,0,1,1,1,1}) \sim \\
(\mathbf{1,1,1,1,1,0,1}) \succ^1 (\mathbf{1,0,1,1,1,0,1}) \succ^1 (\mathbf{1,1,0,1,1,0,1}) \succ^3 (\mathbf{1,1,1,1,0,1,1}) \succ^3 \\
(\mathbf{1,1,1,1,1,1,0}) \succ^3 (\mathbf{1,1,1,0,1,1,1}) \succ^1 (\mathbf{0,1,1,1,1,1,1}) \succ^2 (\mathbf{1,1,0,1,0,1,1}) \sim \\
(\mathbf{1,0,1,1,0,1,1}) \sim (\mathbf{1,1,1,1,0,0,1}) \succ^2 (\mathbf{1,0,1,1,1,1,0}) \sim (\mathbf{1,1,0,1,1,1,0}) \sim \\
(\mathbf{1,1,1,1,1,0,0}) \succ^3 (\mathbf{1,1,1,1,0,1,0}) \succ^2 (\mathbf{1,1,0,0,1,1,1}) \sim (\mathbf{1,0,1,0,1,1,1}) \sim \\
(\mathbf{1,1,1,0,1,0,1}) \succ^3 (\mathbf{1,1,1,0,0,1,1}) \succ^3 (\mathbf{1,1,1,0,1,1,0}) \succ^2 (\mathbf{0,1,0,1,1,1,1}) \sim \\
(\mathbf{0,0,1,1,1,1,1}) \sim (\mathbf{0,1,1,1,1,0,1}) \succ^3 (\mathbf{0,1,1,1,0,1,1}) \succ^3 (\mathbf{0,1,1,1,1,1,0}) \succ^5 \\
(\mathbf{0,1,1,0,1,1,1}) \succ^6 (\mathbf{0,0,0,0,0,0,0})
\end{array}$$

Applying the Algorithm 1, a 3-additive fuzzy measure is identified as the solution of the quadratic optimization program of Tab. II related to the preferences of Tab. V. The non-zero coefficients of the identified 3-additive fuzzy measure are presented in Tab. VI.

As one can see in Tab. VI, from these preferences, the Möbius transform coefficients for the attributes taken alone, that coincide with their fuzzy measure coefficients, are zero. Thus the attributes alone have no importance in the decision. While when the attributes are combined with other ones, some of them become important. Indeed, the Tab. VII shows the corresponding Shapley values and one can see that the attribute ”Climate change” and ”Total energy consumption” have more than 50% of global importance in the decision. For more analysis based on the fuzzy measure, one can take a look at the pair interactions between attributes shown in Tab. VIII. Three positive interactions seem to be important: $\{CC, TEC\}$, $\{CC, RW\}$ and $\{CC, Nox\}$. Therefore, alternative that have

TABLE VI
THE OBTAINED 3-ADDITIVE FUZZY MEASURE.

subset	m_μ	subset	m_μ	subset	m_μ
$\{1,2\}$	0.033	$\{1,3\}$	0.046	$\{1,2,3\}$	-0.033
$\{1,4\}$	0.283	$\{1,2,4\}$	-0.033	$\{3,4\}$	0.004
$\{1,3,4\}$	-0.033	$\{2,3,4\}$	0.04	$\{1,5\}$	0.19
$\{2,5\}$	0.007	$\{1,2,5\}$	0.0001	$\{3,5\}$	0.00714
$\{1,3,5\}$	-0.009	$\{2,3,5\}$	-0.007	$\{4,5\}$	0.055
$\{1,4,5\}$	-0.047	$\{2,4,5\}$	-0.007	$\{3,4,5\}$	0.001
$\{1,6\}$	0.064	$\{1,2,6\}$	-0.025	$\{1,3,6\}$	-0.03
$\{2,3,6\}$	0.127	$\{4,6\}$	0.007	$\{1,4,6\}$	-0.047
$\{2,4,6\}$	0.028	$\{3,4,6\}$	0.015	$\{5,6\}$	0.004
$\{1,5,6\}$	-0.025	$\{2,5,6\}$	-0.004	$\{3,5,6\}$	-0.004
$\{4,5,6\}$	0.014	$\{1,7\}$	0.212	$\{1,2,7\}$	-0.008
$\{1,3,7\}$	-0.009	$\{2,3,7\}$	0.018	$\{4,7\}$	0.023
$\{1,4,7\}$	-0.05	$\{2,4,7\}$	0.011	$\{3,4,7\}$	0.012
$\{5,7\}$	0.003	$\{1,5,7\}$	-0.014	$\{2,5,7\}$	-0.003
$\{3,5,7\}$	-0.003	$\{4,5,7\}$	0.163	$\{1,6,7\}$	-0.004
$\{2,6,7\}$	0.0002	$\{4,6,7\}$	0.022	$\{5,6,7\}$	-0.0001

TABLE VII
THE SHAPLEY VALUES.

CC	AC	Eutro	TEC	RW	RE	Nox
0.293	0.0548	0.0577	0.217	0.1525	0.0606	0.1643

good performances on these pair of attributes will be favored by the DM preferences.

TABLE VIII
THE PAIR INTERACTION VALUES.

	CC	AC	Eutro	TEC	RW	RE	Nox
CC	.	−0.0164114	−0.00979518	0.178723	0.142996	−0.000971418	0.170091
AC	.	.	0.0721558	0.0198734	−0.003587	0.0633496	0.00893116
Eutro	.	.	.	0.0220475	−0.00357115	0.0545434	0.0089312
TEC	.	.	.	.	0.118258	0.0235369	0.102093
RW	.	.	.	.	.	−0.00507813	0.0748298
RE	.	.	.	.	.	.	0.0089312

B. *Value Functions Determination*

In this part, the original MACBETH methodology is used to determine the value functions associated to the LCA impacts expressed in different units of measure in Tab. IV. For each attribute, the DM provided the answers to the MACBETH questioning procedure (see Tab. IX).

From These preferences, we obtain the value function v_i and their aggregation v using the 3- additive fuzzy measure of Tab. VI (see Tab. X). Finally, considering the decision matrix in Tab. IV, the DM preferences on binary alternatives in Tab. V and the DM preferences on fictive alternatives in Tab. IX, we obtain the following ranking: 1: EV DE, 2: EV EU27, 3: EV FR, 4: Dies, 5: Gas.

The most important remark is that the DM considers that on each attribute the result obtained by the best alternative is far, i.e. the difference of attractiveness is ”extreme”, from the level of complete satisfaction. Therefore, all the alternatives have below average satisfactions. The alternatives ”EV DE” obtains almost average value on all the impacts which placed it in the top of the ranking instead of the other alternatives that obtain almost average value in several criteria but a very

TABLE IX
THE DIFFERENCE OF ATTRACTIVENESS MATRICES.

CC	$\mathbf{1}$	$(0.678,\mathbf{1}_{-1})$	$(1.49,\mathbf{1}_{-1})$	$(1.78,\mathbf{1}_{-1})$	$(2.22,\mathbf{1}_{-1})$	$(2.69,\mathbf{1}_{-1})$	$\mathbf{0}$
$\mathbf{1}$	-	extreme	extreme	extreme	extreme	extreme	extreme
$(0.678,\mathbf{1}_{-1})$	-	-	very strong	very strong	extreme	extreme	extreme
$(1.49,\mathbf{1}_{-1})$	-	-	-	moderate	strong	very strong	very strong
$(1.78,\mathbf{1}_{-1})$	-	-	-	-	strong	very strong	very strong
$(2.22,\mathbf{1}_{-1})$	-	-	-	-	-	very strong	very strong
$(2.69,\mathbf{1}_{-1})$	-	-	-	-	-	-	very weak
AC	$\mathbf{1}$	$(34.3,\mathbf{1}_{-2})$	$(41.5,\mathbf{1}_{-2})$	$(47.8,\mathbf{1}_{-2})$	$(49,\mathbf{1}_{-2})$	$(70.3,\mathbf{1}_{-2})$	$\mathbf{0}$
$\mathbf{1}$	-	extreme	extreme	extreme	extreme	extreme	extreme
$(34.3,\mathbf{1}_{-2})$	-	-	moderate	moderate	moderate	strong	strong
$(41.5,\mathbf{1}_{-2})$	-	-	-	strong	strong	strong	strong
$(47.8,\mathbf{1}_{-2})$	-	-	-	-	null	strong	strong
$(49,\mathbf{1}_{-2})$	-	-	-	-	-	strong	strong
$(70.3,\mathbf{1}_{-2})$	-	-	-	-	-	-	very weak
Eutro	$\mathbf{1}$	$(2.56,\mathbf{1}_{-3})$	$(3.75,\mathbf{1}_{-3})$	$(4,\mathbf{1}_{-3})$	$(4.27,\mathbf{1}_{-3})$	$(6.46,\mathbf{1}_{-3})$	$\mathbf{0}$
$\mathbf{1}$	-	extreme	extreme	extreme	extreme	extreme	extreme
$(2.56,\mathbf{1}_{-3})$	-	-	moderate	strong	strong	very strong	very strong
$(3.75,\mathbf{1}_{-3})$	-	-	-	weak	weak	very strong	very strong
$(4,\mathbf{1}_{-3})$	-	-	-	-	weak	strong	strong
$(4.27,\mathbf{1}_{-3})$	-	-	-	-	-	moderate	strong
$(6.46,\mathbf{1}_{-3})$	-	-	-	-	-	-	very weak
TEC	$\mathbf{1}$	$(2.99,\mathbf{1}_{-4})$	$(3.02,\mathbf{1}_{-4})$	$(3.09,\mathbf{1}_{-4})$	$(3.32,\mathbf{1}_{-4})$	$(4.11,\mathbf{1}_{-4})$	$\mathbf{0}$
$\mathbf{1}$	-	extreme	extreme	extreme	extreme	extreme	extreme
$(2.99,\mathbf{1}_{-4})$	-	-	null	very weak	moderate	very strong	very strong
$(3.02,\mathbf{1}_{-4})$	-	-	-	very weak	weak	strong	strong
$(3.09,\mathbf{1}_{-4})$	-	-	-	-	moderate	strong	strong
$(3.32,\mathbf{1}_{-4})$	-	-	-	-	-	strong	strong
$(4.11,\mathbf{1}_{-4})$	-	-	-	-	-	-	very weak
RW	$\mathbf{1}$	$(1.27,\mathbf{1}_{-5})$	$(1.28,\mathbf{1}_{-5})$	$(5.08,\mathbf{1}_{-5})$	$(6.25,\mathbf{1}_{-5})$	$(15.1,\mathbf{1}_{-5})$	$\mathbf{0}$
$\mathbf{1}$	-	very strong	very strong	very strong	extreme	extreme	extreme
$(1.27,\mathbf{1}_{-5})$	-	-	null	strong	strong	very strong	very strong
$(1.28,\mathbf{1}_{-5})$	-	-	-	strong	strong	very strong	very strong
$(5.08,\mathbf{1}_{-5})$	-	-	-	-	weak	strong	strong
$(6.25,\mathbf{1}_{-5})$	-	-	-	-	-	strong	strong
$(15.1,\mathbf{1}_{-5})$	-	-	-	-	-	-	very weak
RE	$\mathbf{1}$	$(7.85,\mathbf{1}_{-6})$	$(7.94,\mathbf{1}_{-6})$	$(10.6,\mathbf{1}_{-6})$	$(12.8,\mathbf{1}_{-6})$	$(23.2,\mathbf{1}_{-6})$	$\mathbf{0}$
$\mathbf{1}$	-	extreme	extreme	extreme	extreme	extreme	extreme
$(7.85,\mathbf{1}_{-6})$	-	-	null	moderate	moderate	strong	strong
$(7.94,\mathbf{1}_{-6})$	-	-	-	moderate	moderate	strong	strong
$(10.6,\mathbf{1}_{-6})$	-	-	-	-	moderate	strong	strong
$(12.8,\mathbf{1}_{-6})$	-	-	-	-	-	strong	strong
$(23.2,\mathbf{1}_{-6})$	-	-	-	-	-	-	very weak
Nox	$\mathbf{1}$	$(14.1,\mathbf{1}_{-7})$	$(20,\mathbf{1}_{-7})$	$(23.4,\mathbf{1}_{-7})$	$(26.6,\mathbf{1}_{-7})$	$(34.8,\mathbf{1}_{-7})$	$\mathbf{0}$
$\mathbf{1}$	-	extreme	extreme	extreme	extreme	extreme	extreme
$(14.1,\mathbf{1}_{-7})$	-	-	moderate	strong	strong	very strong	very strong
$(20,\mathbf{1}_{-7})$	-	-	-	moderate	moderate	strong	strong
$(23.4,\mathbf{1}_{-7})$	-	-	-	-	weak	strong	strong
$(26.6,\mathbf{1}_{-7})$	-	-	-	-	-	strong	strong
$(34.8,\mathbf{1}_{-7})$	-	-	-	-	-	-	very weak

TABLE X
THE DECISION MATRIX.

	EV DE	EV EU27	EV FR	Gas	Dies
v_1	0.358544	0.384354	0.615246	0.0252104	0.230492
v_2	0.384752	0.0002	0.453636	0.454036	0.384752
v_3	0.311975	0.2495	0.4998	0.37445	0.062075
v_4	0.424614	0.424951	0.424814	0.0002	0.289509
v_5	0.364483	0.270633	0.09345	0.635117	0.635117
v_6	0.416421	0.333129	0.0826793	0.4997	0.4997
v_7	0.433021	0.40004	0.4998	0.466419	0.0325813
v	**0.356379**	**0.287885**	**0.285445**	**0.0647806**	**0.143392**

bad result in one or two impacts. The alternative ”Gas” has bad performance on ”CC” and ”TEC” that are the important attributes and that have positive interaction. Consequently, this alternative is ranked last.

V. CONCLUSIONS

In this paper an extension of the MACBETH methodology to non-additive models is proposed. In particular, the questioning and the disaggregation procedures are extended to handle numerous parameters identification for non-additive models. An illustration is given to show how to use this extension in real life application.

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